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需求彈性與不確定性對品牌延伸決策之影響：實質選擇權法

The Impacts of Demand Elasticity and Uncertainty on the Decisions for the Brand Extension: A Real Options Approach

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摘要

有鑑於以往品牌延伸之文獻大多由消費者觀點，運用傳統淨現值法，探討正面品牌延伸效益之影響因素，相對忽略品牌延伸與品牌識別系統之建立，皆屬企業不可回復性之專屬性投資，而且雖有文獻提及品牌延伸進入時點，然而並未詮釋企業面臨不同需求彈性與市場不確定性，如何應用實質選擇權，藉以發揮行銷策略管理彈性。職是之故，本文擴充 Dias 與 Ryals (2002) 之單一階段斷續時間之品牌延伸選擇權，由企業角度建立品牌識別系統與延伸投資決策為之兩階段成長選擇權模型，同時配合比較靜態與敏感度分析，發現延伸品牌之需求價格彈性小於核心品牌需求價格彈性，代表企業對延伸市場之市場力提高，企業從事品牌延伸可以帶來較高的成長選擇權價值。其次，企業等待市場波動帶來最佳投資機會，再選擇最適品牌延伸時點，更能發揮品牌延伸之策略彈性效益。再者，只有在品牌延伸之行銷投資支出所帶來的成長選擇權價值超過投資門檻，企業才會採取品牌延伸策略。另外，核心品牌之行銷經驗愈豐富，愈有利於對延伸產品之營運成本的掌控，對品牌延伸之成長選擇權價值提高大有裨益。

【關鍵字】品牌延伸、實質選擇權、需求彈性

Abstract

Several previous research papers investigate the positive impacts of the brand extension by using the traditional Net Present Value (NPV) approach, which neglects the irreversibility of the specific investments in the brand identification system (BIS) and the brand extension activities. Even though there are some discussions of the optimal entry time for the brand extension in the previous works, it is seldom analyzed that how a company uses the real option approach to deal with the managerial flexibility in marketing strategy under uncertainty. This work extends the model of the single-stage real option for the brand extension in the context of the continuous time (Dias & Ryals, 2002) to the two-stage growth option embedded in the BIS and the brand extension. The comparative static and the sensitivity analyses are performed in this work. The results show that when the positive effects of brand association occur, the demand elasticity for brand extension will be lowered, representing the higher market power in the extended market and the higher value for growth option. Moreover, more market demand volatility encourages the companies to choose the optimal timing for brand extension and bring in more value of the growth option. Furthermore, entering the market too early will cause the investment become irreversible, thus only when the growth of option value derived from the marketing investment expenditures for brand extension excesses the investment threshold, the enterprise will adopt the brand extension strategy. In addition, the marginal operation cost for brand extension is lower than that for BIS stage, and thus increases the growth option value.

【Keywords】brand extension, real option, demand elasticity

I. Introduction

To respond to customers' increasing consciousness and to cope with strong competition in the industry environment, any company facing great market uncertainties should consider the strategies of new brand or brand extension to strengthen its competitiveness. Hitsch (2006) pointed out that the company introducing a new brand requires great advertisement expenditures, faces the greater uncertainty of revenues and profits, and therefore suffers from high failure rate of 70%. Conversely, the line extension, which adopts the common family brand, and the category extension, which switches to different product lines, are more successfully accepted and implemented.

Keller (1991) pointed out that 63% of the new products belong to line extension, while 18% of them belong to category extension. Aaker (1991) indicated that 89% of the companies which introduce the new consumer products, adopt the line extension, 6% of them adopt the category extension, and only 5% of them adopt the new brand strategy. It is therefore obvious that the majority of the companies introducing new product adopt the line extension strategy.

Even though the brand extension brings in many tangible and intangible benefits, this research advocates that the valuation of the brand extension by using the traditional Net Present Value (NPV) approach is unable to discover the true value of the brand extension in terms of irreversibility for those companies with limited resources. Once the strategic investment of the marketing expenditures fails to survive, the company will incur the high switching costs or the sunk costs. A company, which considers both the market uncertainty and the demand elasticity, will make a better brand extension decision in terms of the managerial flexibility.

Sullivan (1992), Reddy, Holak, and Bhat (1994), Kotler (1999), Pattikawa (2006) all pointed out that the timing of the brand extension is very critical. However, the early implementers do not necessarily have the first mover advantage because of the inconsistent customers' preferences and the volatile changes in market conditions. Moreover, according to the arguments of real options proposed by Kogut and Kulatilaka (1994), Trigeorgis (1995), and Brennan and Trigeorgis (2000), the optimal time of the brand extension stochastically exits. Therefore, the companies should cope with the market uncertainties, maintain the flexibility in their operations, and adjust their managerial strategies to grasp the best investment opportunity.

Dias and Ryals (2002) pointed out that the right timing of the brand extension, which is based on the competitiveness of the existing core brand, will bring in the value of the future

growth for companies. That is, the net present value of the brand extension combined with the value of the real option comprises of the value of the brand equity. In view of the underestimate of the brand equity by using traditional NPV, which neglects the effects of managerial flexibility, this work advocates that the brand equity should consist of the establishments of the brand identification system (BIS) and those of the brand extension. In such a context, the dynamics of the value of the growth option could be correctly valuated.

This work is based on the two-stage sequential real option approach (ROA). A company can optimally schedule the quantities to be delivered to pursuit the maximum profits in respond to the greater uncertainties in a new niche market through the brand extension. Therefore, the process of the investment decisions consists of the following two stages: the first one is the investment decision for BIS, and the second one is the decision for the brand extension. Through the comparative static and the sensitivity analyses, the impacts of the market uncertainties, the demand elasticity, and the costs of the strategic investments on the brand extension decisions could be clearly understood.

II. Literature Review

A. The Establishment of the BIS

The brand origins from the customers' total experiences of goods or services. Aaker (1991) pointed out that a successful brand attributes to its unique brand personality, brand symbol, brand-customer relationship, self-expressive benefit, emotional benefit, user image, original manufacturer country image, and organizational associations, etc. In short, the brand is the source of competitiveness for the company. The customers' associations between the brand and the product, the business, the person, and the symbol through the BIS will increase their faiths in and affiliations with the brand. Therefore, most of the companies think of the strengthening of the "fundamental recognition" and "extended recognition" through the core factors, such as the belief and the value of a brand, the quality of products or services, the personality of the brand, the symbol of the brand, the ritual of the brand, the disciples of the brand, the affiliation of the brand, and the management and the employees, etc [1] (Aaker, 1996).

The establishment of the BIS is a proprietary strategic investment on intangible asset, which is similar to the transformation from the complex thoughts to the simplified language

1 The fundamental recognition of a brand is the essence of a brand and it will nor disappear with time, while the extended recognition is the part which will raise the completeness, richness of a brand and allow the owner of the brand to have more opportunities to grow.

by a good novel or a good poetry. When the BIS provide the customers with the sufficient and guaranteed information to increase the brand position and the customer loyalty, it can also prevent the companies from the redundant marketing expenditures and then arrive at the benefits of scaled economy. However, the investment on BIS is irreversible. When facing with the great uncertainty in a market, the companies should strategically think of the integration of the resources and the managerial flexibility to make best use of the growth opportunities in the future.

B. The Benefits and Risks of the Brand Extension

Diversification is the main route to grow for many companies. For example, market differentiation, "vertical brand extension" with new product development, "horizontal brand extension" with entry to a different product line by the existing common brand , or "multi-brand extension" with a single product line (e.g. GM introduced Chevrolet, Pontiac, Oldsmobile, Buick, Cadilac...etc.), and even "cross-industry brand extension" (e.g. China Wahaha group introduced the juice soft drinks, coke, and dressing). The benefits of the brand extension can be achieved by three folds as follows. Firstly, the introduction of a series of products under the existing core brand to provide the customers with multiple choices, to reduce the customers' perceived risks of the brand extension, and to increase the acceptability of the brand extension. Secondly, the brand extension cuts the operational costs and strengthens the compositeness of the brand. The brand portfolios with multi brands are managed by the brand managers. In this context, the competing environments can be built to promote the positive "effects of the fish". Thirdly, the brand extension can reduce the expenditures in channel marketing and promotion activities to decrease the probability of failure.

The brand extension linked to the existing core brands could be utilized to facilitate the product innovation. Even though the strengthened value of the brand could curb and prevent from the fights with the rivals, and then raise the entry barrier, the value creation of the brand leverage derived from the brand extension (Aaker, 2003) also faces the unpredictable changing environments, because the customers may have the selective notice, selective distortion, and selective reservation. If the product quality of the existing core brand creates the trusts and good feelings for the customers, or if the brand extension is highly correlated with the belief or the personality of the existing brand, the risks of the brand extension will be reduced greatly. In view of this, the companies which implement the brand extension strategies should have a clear understanding of the facts that the strategic investment and the

BIS establishments are irreversible. They should be aware of the uncertainties of demands in their identified niche market and the deterioration of the market shares and the dilution of the recognition of the existing core brands (Loken & John, 1993). Therefore, a company should make best use of the growth opportunities in the niche market, flexibly and efficiently allocate the limited resources, and then maximize the value of the brand extension.

C. The Exploration of the Factors Influencing the Brand Extension

In previous researches on the factors influencing the brand extension, many scholars emphasized not only that the new products through the brand extension should have high similarity with the existing products in product attributes, but also that the concepts of the core brand and its brand extension should comply with each other (Park, Milberg, & Lawson, 1991). Even more, the brand extension had better induce the customer into the brand specific association (Broniarczyk & Alba, 1994). That is, the more strength the existing core brand has, the more similarity with the existing brand the extension brand has, the more expenditures the advertisements and the promotion campaign spend, the more benefits the brand extension generates (Hardie, 1994).

Reddy et al. (1994) pointed out that the factors of the successful line extension include the strength and the value of the existing brand, the entry time of the brand extension, the size of the business, and the capability of the marketing campaign. Kotler (1999) also emphasized that the categories of the brand extension should be matched, the entry time and the means of the brand extension should be grasped properly, and the reasons of the brand extension should be justified. Regarding the entry time of the brand extension, Sullivan (1992) pointed out that the early brand extension in a young market can utilize the existing customer information from the existing brand and the strength of the existing market share. But the negative impacts of the brand extension on the existing brand equity may be derived from the difficulties of the market positioning and the risks of failure. In this case, it is worthwhile to adopt new product lines by the existing core brand names or to defer the brand extension instead of immediate action.

D. The Relationship between the Brand Extension and the Real Option

Most of the companies valuate the BIS and the brand extension by the traditional NPV approach, which assumes that the investment of the brand is reversible. It does not comply with the situation in practice, because the investment of the brand could be a kind of the

sunk costs. Moreover, the NPV approach requires the immediate decision (now or never), which neglects the value of the flexibility (Dixit & Pindyck, 1994). In the context of more uncertainties in a market, many decision makers prefer waiting for the arrival of the new information to grasp the optimal investment opportunities (Kogut & Kulatilaka, 1994; Trigeorgis, 1995; Brennan & Trigeorgis, 2000).

Regarding the valuation in the environments with high risks, Courtney, Kirkland, and Viguerie (1997) pointed out that the decisions for capital investment could be based on the break-even analysis, sensitivity analysis, alternating scenario analysis, statistics distribution analysis, agent-based simulation models, decision tree, game theory, and system dynamics in order to understand the interactions in a market. When facing the changing world, the decision makers are suggested to use the real option approach to deal with the valuation problems of the dynamic investment opportunities[2]. Sudarsanam, Sorwar, and Marr (2006) pointed out that more factors of the uncertainties will lead to more volatile value of the real option.

In spite of the emphasis on the timing problems of the brand extension in the previous researches by many scholars, Dias and Ryals (2002) are the first persons to explain the value creation process of the brand extension by the real option approach. They proposed that the development of the new brands and the creation of the brand equity require large capital expenditures (e.g. advertisement, promotion campaign, and channel marketing costs), which are irreversible but can create the growth opportunities for the companies in the future. Therefore, the companies which want to properly valuate the brand equity should take into account the cost/benefit and the growth opportunities from the brand extension. However, Dias and Ryals (2002) derived their model by a single stage model on the discrete time basis[3], and only described the simplified relationship between the brand extension and the growth opportunities. They did not explore the sequential two-stage investment decisions for both the BIS and the brand extension.

Pattikawa (2006), who agreed with the arguments of Dias and Ryals (2002), pointed out that the decisions of the brand extension should be considered as a kind of call option in order to respond to the volatile stock prices and the uncertainties due to the risks of the

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2 The discrete time has the advantage of easy understanding and handling. Merton (1990) and Dixit and Pindyck (1994) pointed out that the continuous time model is the limits of its discrete time version. Moreover, the continuous time model is both intuitively appealing and quite powerful.

3 According to the research of Copeland (2004), 27% of the CFOs in the USA adopted the real option as a tool of strategic investment valuation.

competition and financial resources among different companies. Pattikawa(2006) statistically tested the brand extension events for 428 chemical companies over the period from 1973 through 2005 by using the Cox survival model. The results verify that the more volatile the company's stock price, the more competitors and the more resource limitations the company faces, the greater uncertainty it faces. To better use the marketing resources and save the costs, early implementation of the brand extension will help maintain the managerial flexibility.

E. Single-stage and Two-stage Real Option Approach

The previous researches by the real option approach are classified into single-stage and two-stage cases. They thought of the growth opportunities in the future as a growth real option, which can be exercised before or at the end of the project and can be regarded as an American call option. The holder of such an option owns the deferral rights and can wait for the optimal entry time at the cost of the exercise price.

In the single-stage case, McDonald and Siegel (1986), Paddock, Siegel, and Smith (1988) assumed that the value and the costs of the investment project follow the geometric Brownian motion (GBM) in a continuous time context. They emphasized that investment costs are irreversible under uncertainties, and the waiting has the flexibility value in the defer option. Kester (1984) pointed out that companies are willing to invest to create the value of the growth option in their production expansion, new product introduction because of the growth opportunities in the future. Kulatilaka and Perotti (1998) discussed that companies can create the strategic value of the growth option with each others in the complete competition and incomplete competition markets.

In the two-stage case, Dixit and Pindyck (1994), and Bar-Ilan and Strange (1998) pointed out that most of the projects in practice, such as oil exploration, introduction of the new medicines, and the development of the aerospace, etc., belong to multi stage and continuous investment projects. The companies should closely watch the market information and the related indexes to suspend the projects when the market faces the downturn trend, or to decide whether to invest at the next stage to avoid the great losses of the sunk costs.

The points of Trigeorgis (1993) on the two-stage growth option regarded the managerial flexibility as a process of value creation by a series of options. Therefore, if the interactions among the multiple options are ignored in valuation, the project value will be underestimated. Smit and Trigeorgis (2004) pointed out that most of the strategic investment opportunities should include the path-dependent continuous projects. Otherwise, the single-

stage option value can not catch the whole potential growth opportunities in the future.

F. The Model of the Two-stage Real Option: the BIS and the Brand Extension

Compared with the single-stage real option model for the brand extension proposed by Dias and Ryals (2002), this work advocates that once the niche market has been identified, the companies which seek for the optimal growth opportunities in the future should establish the BIS by the strategic investment at the first stage, and then adopt the brand extension strategy at the second stage in order to bring in the continuous revenues. The improper implementation of the brand extension could cause the negative dilution and confusion of the brands under the market uncertainties. It also causes the discrepancies between the single-stage and the two-stage options in terms of the contingent growth options.

This work builds the options for the BIS and the brand extension in the context of two-staged process on a continuous time basis, which is dissimilar with that proposed by Dixit and Pindyck (1994). Therefore, this work extends the model of Dias and Ryals (2002) into the two-stage scenario: the BIS and the brand extension.

III. A two-stage sequential ROA model

A. Basic Assumptions

Suppose that a firm faces deterministic inverse demand function for its core brand in the target market at the first stage:

$$P_m = Q_m^{-\frac{1}{\varepsilon_m}} \tag{1}$$

where P_m is the product price for the core brand, Q_m is the quantity of product sold, and ε_m is the demand elasticity for the core brand.

At the second stage, the firm extends its core brand with new demand elasticity and uncertainties:

$$P_s = Q_s^{-\frac{1}{\varepsilon_s}} \cdot Y \tag{2}$$

where P_s denotes the product price for brand extension and it is assumed to be uncertain due to the unknown customers' tastes in a newly identified niche market. Q_s is the quantity of product sold, and ε_s is the demand elasticity for the brand extension.

When we take into account the positive and negative effects of brand association and brand dilution, respectively, the demand elasticity could be used to explain those impacts for both the BIS and the brand extension stages. When the niche market is properly segmented

and the related marketing campaigns are successfully implemented, the company benefits from the positive effects of brand association and possess higher market power in the extended niche market than in its original core brand market, where it faces the declining phase in a product life cycle. In such a situation, the demand elasticity for the brand extension stage should be less than that for the BIS stage ($\varepsilon_m > \varepsilon_s$), because the market power is measured by a function of the demand elasticity as $f(\varepsilon) = \frac{1}{\varepsilon}$. In contrast, when the brand dilution occurs due to the improper product positioning at the brand extension stage, its negative effect causes the company lose the market power at the second stage. In such a situation, the demand elasticity for the brand extension stage could be greater than that for the BIS stage ($\varepsilon_m < \varepsilon_s$). Otherwise, if the brand extension at the second stage brings in no impact at all, the equality $\varepsilon_m = \varepsilon_s$ should be satisfied.

Suppose that the taste of new customers in the extended niche market is idiosyncratic and uncertain. It is denoted by and follows an exogenously geometric Brownian motion process as follows:

$$\frac{dY}{Y} = (r-\delta)dt + \sigma dz . \tag{3}$$

where r is a risk-free rate, δ is a dividend yield, and σ is the standard deviation of Y in unit time interval, while dz is an increment of the Wiener process.

The firm aims to choose Q_m and Q_s to maximize its total profits, denoted by π, for those two stages. We denote the marginal marketing expenditures as w_s and w_m for the BIS and the brand extension stages, respectively. Concerning the similarity between the core brand product and the brand extension product, the marketing expenditures incurred for brand extension are negatively correlated with the degree of similarity in product attributes. The greater the similarity, the smaller the marketing expenditure for brand extension due to the experiences in the marketing campaign for the BIS and the acceptance from existing customers. Therefore, we can set the relationship between w_s and w_m as $w_s = k \cdot w_m$. When those two product lines have very similar attributes, k is less than 1, while they have very different attributes, k is greater than 1. Otherwise, k=1 represents that they could be the same product.

$$\underset{Q_m, Q_s}{Max}\ \pi(P_m, P_s, Q_m, Q_s, Y; w_s, w_m)$$

$$= \underset{Q_m, Q_s}{Max}(P_m \cdot Q_m + P_s \cdot Q_s \cdot Y - (w_s Q_s + w_m Q_m))$$

$$= \underset{Q_m, Q_s}{Max}(Q_m^{1-\frac{1}{\varepsilon_m}} + Q_s^{1-\frac{1}{\varepsilon_s}} \cdot Y - (kw_m Q_s + w_m Q_m)) \tag{4}$$

The first-order necessary conditions for the profit-maximization problem in equation (4) are as follows:

$$\frac{\partial \pi}{\partial Q_m} = (1-\frac{1}{\varepsilon_m}) Q_m^{-\frac{1}{\varepsilon_m}} - w_m = 0 \quad , \tag{5}$$

and $$\frac{\partial \pi}{\partial Q_s} = (1-\frac{1}{\varepsilon_s}) Q_s^{-\frac{1}{\varepsilon_s}} Y - kw_m = 0 . \tag{6}$$

From equations (5) and (6), we can derive optimal quantities sold for the core brand and the brand extension, respectively, as follows:

$$Q_m^* = \left(\frac{1-\frac{1}{\varepsilon_m}}{w_m} \right)^{\varepsilon_m} , \tag{7}$$

and $$Q_s^* = \left(\frac{(1-\frac{1}{\varepsilon_s})Y}{kw_m} \right)^{\varepsilon_s} . \tag{8}$$

Substituting equations (7) and (8) into the profit function, we can obtain the optimal profits: $\pi^* = A + B \cdot Y^{\varepsilon_s}$ (9)

where

$$A = \frac{1}{\varepsilon_m} \left[\frac{1-\frac{1}{\varepsilon_m}}{w_m} \right]^{\varepsilon_m - 1} \tag{10}$$

$$B = \frac{1}{\varepsilon_s} \left[\frac{1-\frac{1}{\varepsilon_s}}{kw_m} \right]^{\varepsilon_s - 1} . \tag{11}$$

Equation (9) indicates that the total profits from both the BIS and the brand extension stages are affected by the movements in the idiosyncratic taste and other factors, such as marginal operation cost w_m and w_s, demand elasticity (market power) ε_m and ε_s.[4]

[4] Lerner's degree of monopoly power is defined as $0 < M_L = \frac{P-MC}{P} = \frac{1}{\varepsilon} < 1$, where ε is the elasticity of demand: $MR=MC=P(1-1/\varepsilon)$.

B. The value of brand investment-extension projects

Dixit and Pindyck (1994) pointed out that the uncertainties in the real option come from the value and the price of the project. When a company makes decisions for the investment project, it is assumed that the production quantity is fixed at one unit in their model. Based on the sunk costs, the optimal threshold of the investment can be derived. But this work assumes that the quantity delivered is varied instead of being fixed. The optimal delivery quantities (Q_m , Q_S) are determined first, and then the project value, V (Y), is derived. Combining the Lemma 1 and Lemma 2, the value of the growth option form the BIS, F_1 (Y), and the value of the growth option from the brand extension, F_2 (Y), can be sequentially derived.

Lemma 1: Suppose that demand risks for both the BIS and the brand extension can be hedged completely through capital markets. Then V (Y) satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 Y^2 V''(Y) + (r-\delta)YV'(Y) - rV(Y) + \pi^* = 0, \tag{12}$$

Proof: See Appendix 1.

Lemma 2: The value of brand investment-extension project is

$$V(Y) = \frac{A}{r} + \frac{B\,Y^{\varepsilon_s}}{\Delta} \tag{13}$$

where

$$\Delta = r - (r - \delta - \frac{1}{2}\sigma^2)\varepsilon_s - \frac{1}{2}\sigma^2\varepsilon_s^{\,2} > 0 \tag{14}$$

Proof:: See Appendix 2.

From equations (13) and (14), the value of the core brand and the brand extension,V (Y), consists of two parts[5] : (1) the discounted profit generated from the BIS investment at the first stage, which equals the net present value of the cash flows discounted by the risk free rate, and (2) the uncertain profit generated from the brand extension at the second stage, which is jointly determined by the risk free rate (r), the dividend yield rate (δ), the demand elasticity in the extended market, and the adjusted discount rate (Δ). Based on the value,V (Y), of the BIS and the brand extension, and the value (F_2 (Y)) of the growth option for the

5 In the theory of the single-stage real option, Dixit and Pindyck (1994) derived the project value by the discounted net present value of the uncertain profits in the future.

brand extension, the investment threshold (Y_2^*) is derived, and then those values (F_1 (Y),Y_1^*) for the BIS at the first stage are backwards obtained.

C. The option value of brand extension

Define the value of brand extension option, F_2 (Y), as follows:

$$F_2(Y) = \underset{t_2 \in [T_1, \infty)}{Max} E_{T_1}\left\{\left[V(Y) - I_2, 0\right] e^{-r(t_2 - T_1)}\right\} \quad , \tag{15}$$

where E_{T_1} is the conditional expectation operator at time T_1 , $V(Y)$ is the combined value for both the BIS and the brand extension stages, t_2 is the entry time for the brand extension, and T_2 is the maturity date at which the firm stops its brand extension.

Equation (15) indicates that when the discounted present value of the brand extension at the cost of sunken expenditures, denoted by, I_2 is greater than the threshold value of demand shock level, the firm will start to extend. If a company implements its strategy of the brand extension in an extended market with uncertain demands, the dynamic programming approach can be used to calculate the value, $F_2(Y)$, of the growth option for the brand extension.

Proposition 1: The growth option value of the brand extension is given by

$$F_2(Y) = D_2 \cdot Y^{\beta_2}, \tag{16}$$

where

$$D_2 = \frac{\varepsilon_s}{(\beta_2 - \varepsilon_s)}\left(I_2 - \frac{A}{r}\right)(Y_2^*)^{-\beta_2} > 0 \quad , \tag{17}$$

$$Y_2^* = \left[\frac{(I_2 - A/r)\Delta\beta_2}{B(\beta_2 - \varepsilon_s)}\right]^{\frac{1}{\varepsilon_s}} \quad , \tag{18}$$

and $$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{(r-\delta)}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}\,. \tag{19}$$

Proof: See Appendix 3.

When the actual demand shock level exceeds its optimal threshold, $Y \geq Y_2^*$, the firm exercises its extension option. Equations (17), (18), and (19) reveal that the value of extension option $F_2(Y)$ is greater than zero. This implies that the traditional net present value approach underestimates the optimal threshold of investment, because it ignores the value of

flexibility when facing uncertainty of investment with sunk costs.

D. The option value of the BIS brand investment

Givenand $F_2(Y)$ and Y_2^* derived in the above section for the firm's second-stage problem, we proceed to calculate the value of the option for the BIS brand investment, $F_1(Y)$, and its optimal threshold value, denoted by Y_1^* , for the first stage. The value of the option for the BIS brand investment can be written as follows:

$$F_1\left(F_2(Y)\right) = \underset{t_1 \in [0,\, T_1]}{Max} E_0\left\{\left[F_2\left(Y\right) - I_1, 0\right] e^{-rt_1}\right\}, \tag{20}$$

where E_0 is the conditional expectation operator at time 0, t_1 is the entry time of the BIS investment, T_1 is the time at which the firm stops the BIS investment, and I_1 is the sunk cost of the BIS brand investment.

Proposition 2: The growth option value of the brand investment is given by

$$F_1(Y) = D_1 \cdot Y^{\beta_1}. \tag{21}$$

where

$$D_1 = \frac{D_2 \beta_2}{\beta_1} \cdot (Y_1^*)^{\beta_2 - \beta_1}, \tag{22}$$

$$Y_1^* = \left[\frac{\beta_1 I_1}{(\beta_1 - \beta_2) D_2}\right]^{\frac{1}{\beta_2}}, \tag{23}$$

$$\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{(r-\delta)}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{24}$$

Proof: See Appendix4.

Compared with the model for the single-stage option value proposed by Dias and Ryals (2002), this work integrates the uncertainty and the irreversibility of brand investment to valuate the growth option, $F_1(Y)$, for the BIS investment and the growth option, $F_2(Y)$, for the brand extension.

From propositions 1 and 2, the entry thresholds (Y_1^* and Y_2^*) and the value of the growth options ($F_1(Y)$, $F_2(Y)$) for both the BIS and the brand extension are affected by the demand shock volatility (σ), the different demand elasticity (ε_s , ε_m), the sunken marketing expenditures for the BIS (I_1) and the brand extension (I_2) stages. Among those factors, the future demand shock volatility and the sunken expenditures for the brand extension are analogous to the volatility of the stock price and the uncertainty of the financial resource

limitations in the empirical studies of Pattikawa (2006). The demand elasticity, which captures the customers' sensitivity of demand to the price, is not mentioned in the previous studies.

IV. Comparative statics and sensitivity analysis

To provide readers with more evidences in practice, we collected the marketing expenditures for two listed companies in food industry in Taiwan: AGV product corporation and Hey-song corporation. According to exhibit 1, AGV introduced its famous and popular tomato drink in 2002 as a core brand. It extended its core brand to milky products in 2004, such as Goyuan Pure Milk and Goyuan Yogurt Milk, etc. Hey-song corporation introduced its core brand Justea and the brand extension Collagen Yogurt Milk in 2003 and in 2005, respectively. The related marketing expenditures for both the sunk and the operational costs are also provided in exhibit 1 and 2 according to their financial statements and reports.

Those data could be used for the actual value for the parameters in our framework for further analyses. It can be evidently shown that companies in Taiwan indeed separate their branding strategies into two stages as stated in our model. Such a two-stage strategy allows the companies to obtain more flexibility of product introductions. Those companies who adopt the two-stage strategy for their brand extension have rights to excise the growth option in response to the favorable market trends or to suspend the brand extension when the market condition is unfavorable. For example, the entry time for the brand extension is dynamically dependent on the customers' responses to the core brand and the prospects for the extended market. Once the consumers change their preferences, the impacts of such changes will be propagated into the changes in demand elasticity, which in turn alter the optimal entry decisions for the brand extension. However, the marketing campaigns could be implemented to induce the customers to go on the favorable way at the cost of sunken expenditures, such as advertisement, promotion program, and public relations expenditures, etc.

Exhibit 1. The two stages of brand extension and the marketing expenditures for AGV product corp.

Time	Stage	Product Introduction	Sunk Cost (NT$,000)
Sep,2002	Core Brand Establishment	AGV Honey Tomato Drink	\$58,707 (I_1)
Sep,2004	Brand Extension Establishment	AGV Milky Product	\$142,572 (I_2)

Exhibit 2. The two stages of brand extension and the marketing expenditures for Hey-song corp.

Time	Stage	Product Introduction	Sunk Cost (NT$,000)
Jun,2003	Core Brand Establishment	Hey-song Justea	\$38,846 (I_1)
Jun,2005	Brand Extension Establishment	Hey-song Collagen Yogurt Milk	\$95,530 (I_2)

To explore the impacts of the brand extension decision on the value of the growth option, this work derives the analytical solution to the growth option value for both the BIS and the brand extension. Subsequently, the sensitivity of the demand volatility (σ) , investment costs of the brand extension (I_2) , and demand elasticity (ε_s) to the value of the growth option ($F_2(Y)$) is performed. Besides, to compare with the results in the two-stage option model proposed by Dixit and Pindyck (1994), we follow the parameter sets in their work. For example, the volatility of the stock price is set to σ=0.2 , the risk free rate is set to r=0.04, the dividend yield is δ=0.04 . Based on the above parameters, the solution to equation (19) is β_2=2 .

From equations (16) and (17) in proposition 1, the value of the two-stage growth option is determined by the market demand elasticity (ε_m , ε_s), β_2 , σ , r , and δ . To perform the sensitivity analysis of the growth option in the brand extension, besides demand volatility (σ) and investment cost (I_2) of the brand extension in the model of Dixit and Pindyck (1994), the market demand elasticity (ε_s) should be considered. To explain the impacts of the demand elasticity and the investment costs of the brand extension on the growth option, the demand elasticity is set to ε_m = ε_s=1.5 [6], the other manufacturing cost is set to k=1 and w_m =1 , and the marketing expenditures spent in the brand extension is set to I_2 =20 .

A. The effect of elasticity of demand on growth option value of brand extension

The price elasticity of demand is used to measure the responsiveness of the quantity demanded of a good to be changed in its price. The factors, affecting the price elasticity of demand, come mainly from the customers' preference, income, prices of substitute goods (Nicolson, 2004). If a company faces the greater price elasticity of demand in the extended market, it requires the strengths of the well-known brand name, which was established in the

6 It is assumed that the demand elasticity is greater than 1. Otherwise, the marginal revenue will be less than 0.

BIS. The early involvements of the line extension or the category extension will bring in the more growth opportunities in the extended market and the more value of the managerial flexibility. As we stated earlier, the demand elasticity for brand extension is greater than that for the core brand due to the effect of brand dilution ($\varepsilon_m \leq \varepsilon_s$). We therefore set to 1.5 and ε_s from 1.5 through 1.9 under the parameters set β_2=2, I_2=20 , and w_s =w_m =1 .

Tauber (1988) found that when customers recognize the consistency between core brand and brand extension, brand extension is more likely to survive. That is, recognition fit is a key factor for the success of brand extension. This paper advocates that the more fits in product personality, which is transferable in technical (internal fit) (Aaker & Keller, 1990; Park et al., 1991), in the recognition consistency for brand concept (Dawar & Anderson, 1994), in the replacement or compliment of products in usage context (Aaker & Keller, 1990), and more fits in the transfer of brand association (Broniarczyk & Alba, 1994) (external fit) (Smith & Park, 1992) all contribute to the positive effects of brand association on core brand and on brand extension as well. For example, the positive evaluation of customers for NIKE sporting shoes is more likely to cause the positive association for NIKE sports suites. In contrast, the inconsistency evaluation of customers for the product personality or brand concept will lessen the degree of their evaluation for brand extension and cause the negative effects of brand dilution (Boush & Loken, 1991; Loken & John, 1993; Bhat & Reddy, 1997). For example, customers are unlikely to accept the brand extension from Harley motors to perfume.

Different from the previous works, this paper integrates the demand elasticity at two different stages into a real option framework, and allows for the flexible strategy for brand extension. In short, the more fit between core brand and brand extension, the lower the customer recognition risks, the higher the customer evaluation for brand extension. When the positive effects of brand association occur, the demand elasticity for brand extension will be lowered ($\varepsilon_m \geq \varepsilon_s$), representing the higher market power in the extended market and the higher value for growth option. For example, when ε_s is set to 1.5, increasing ε_m from 1.5 to 1.9 will decrease the growth option value form 17.26 to 15.89 (see Exhibit 3 and Figure 1).

Exhibit 3 Sensitivity of growth option value $F_2(Y)$ to demand elasticity ε_m under ε_s=1.5 when the effect of brand association occurs.

Demand elasticity ε_m	1.5	1.6	1.7	1.8	1.9
growth option value $F_2(Y)$	17.26	16.76	16.40	16.12	15.89

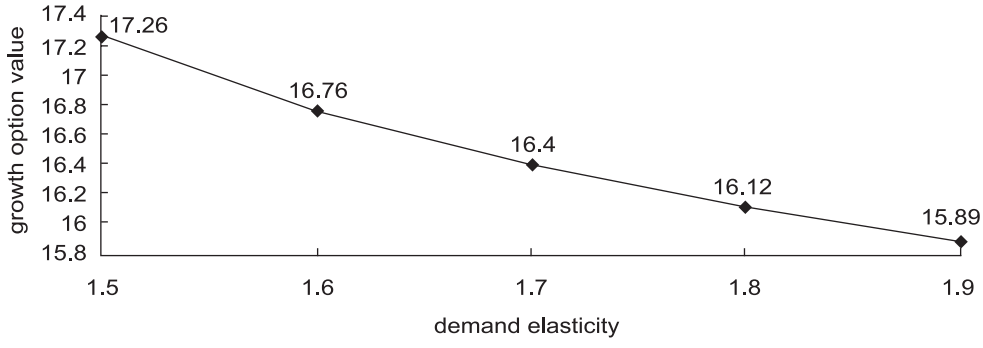

Figure 1 Sensitivity of growth option value under the brand association

The improper practice of brand extension could be harmful to the value of the core brand (Ries & Trout, 1986). This paper advocates that the inconsistent fit between the core brand and brand extension results in the worse images of the core brand, and subsequently damages the brand association (Morrin, 1999). As a result, the demand elasticity for brand extension will be raised ($\varepsilon_m \leq \varepsilon_s$), representing the decreases in market power. In such a condition, the growth option value will be decreased. For example, from exhibit 4 and figure 2, when ε_s is set to 1.5, increasing ε_m from 1.1 to 1.5 will decrease the growth option value, $F_2(Y)$, form 29.31 to 17.26.

Exhibit 4 Sensitivity analysis of growth option value $F_2(Y)$ to demand elasticity ε_m under ε_s =1.5 when the effect of brand dilution occurs.

Demand elasticity ε_m	1.1	1.2	1.3	1.4	1.5
growth option value $F_2(Y)$	29.31	21.40	19.13	17.98	17.26

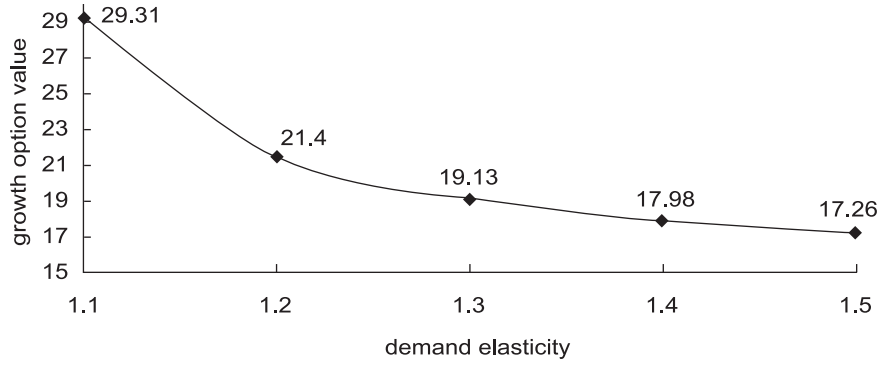

Figure 2 Sensitivity of growth option value under the brand dilution

Based on the above results, we present proposition 3 as follows.

Proposition 3: The greater the demand elasticity for brand extension, the smaller the market power, the less the growth option value for brand extension. That is, $\frac{\partial F_2(Y)}{\partial \varepsilon_s} < 0$.

B. The effect of market demand volatility on growth option value of brand extension

Facing the market demand uncertainty in the future, the specific investment expenditures of the brand are irreversible. If a strong brand can be established by the company, it will contribute to the growth opportunities form the brand extension in the future (Kester, 1984; Kulatilaka & Perotti, 1998).

This work finds that if a company forecasts the trends for the economic development in the future in a more opportunistic way and it is willing to assume higher risks in the two-stage scenario, the more volatile market demand will encourage the company to take the strategy of the brand extension. As a result, the growth option value is raised.

Proposition 4: The higher the demand volatility, the more valuable the growth option for brand extension, that is $\frac{\partial F_2(Y)}{\partial \sigma} > 0$.

Proof: See Appendix 5.

Because a firm undertakes the projects for core brand and brand extension at different time, it faces the different demand risks. Generally speaking, when a firm extends its core brand at the early stage to take benefits of the existing customer information and of the first mover advantage, it faces high risks of failures, because the customers' preferences are still difficult to capture, and the brand extension is difficult to position. As a result, the firm gains the first mover advantage at the cost of more volatile brand extension and high failure probability. In order to overcome such pitfalls, it should closely and intensely monitor the uncertainty and volatility in market demands, wait for the emergence of uptrend in market demand and find the optimal entry time for the brand extension.

Even though larger volatility could bring in more downside risks, we can avoid its negative effects and take the greater growth potential by waiting for the emergence of reversal uptrend. In doing so, waiting for favorable conditions and grasping larger growth opportunities make the growth option more valuable. Marketing campaigns could be implemented by marketers to transform the negative effects caused by larger volatility into

greater growth opportunities. From exhibit 5 and figure 3, when the demand volatility increases form 0.18 to 0.22, the value of the growth option for the brand extension also increases form 9.15 to 18.30. The results show that when the market demand volatility is greater, the customers can have more free choices of the brand. A company can obtain the greater value of the growth option only through the adequate adoption of the brand extension, basing its competitiveness on the strong existing brands and the information share rate. This also confirms the arguments of Sullivan (1992).

Exhibit 5. Sensitivity of growth option value $F_2(Y)$ to market demand volatility σ

market demand volatility σ	0.18	0.19	0.2	0.21	0.22
growth option value $F_2(Y)$	9.15	10.84	12.85	15.29	18.30

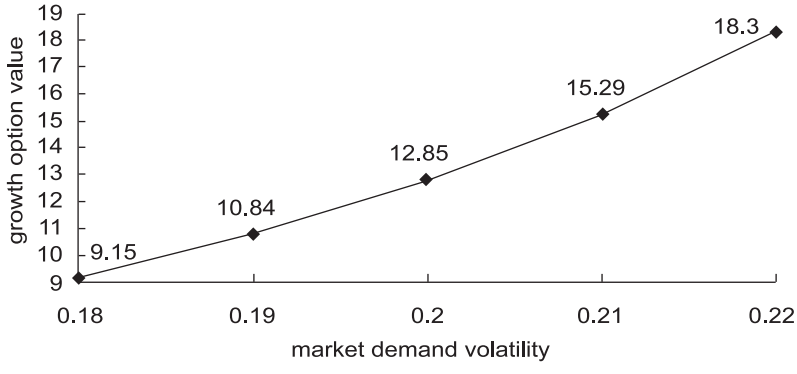

Figure 3. Sensitivity of growth option value to market demand volatility

C. The effect of marketing investment on growth option value of brand extension

The brand with scarcity, value, specificity, and necessity, is an intangible asset for a company. Many companies intend to increase its brand equity through the strategic brand investment. After the establishment of the BIS, the succeeding tasks, including the brand extension, the marketing campaign, the channel creation, the resource commitment, the well-known brand name, the perceived quality of the product, the brand association, and the brand loyalty, all affect the brand equity and the competitiveness for a company.

Proposition 5: The greater the marketing expenditure on brand extension, the lower the growth option value. That is $\dfrac{\partial F_2(Y)}{\partial I_2} < 0$.

Decisions for brand extension involve the important strategic growth goals for

enterprises. Once the wrong decision occurs, the company not only has a great deal of loss of time and resources, but also loses the investment opportunities to other markets. Especially, to do the brand extension, the extra brand marketing expenditures are needed as well as the marketing channel costs. Those strategic marketing investment can not be depreciated. However, in order to have more growth opportunities in the future, the enterprises have no choice but to invest the irreversible resources; thus, they are used to choose the entering timing carefully when they decide to have the investment.

From the result of exhibit 6 and figure 4, a company, which incurs more costs of the brand extension on the specific brand asset (e.g. increases from 18 units to 26 units), will decrease growth opportunities for its growth option. For example, the option value for the brand extension decreases from 18.53 to 14.82. This outcome reveals that only when the growth of option value derived from the marketing investment costs for brand extension excesses the investment threshold, the enterprise will adopt the brand extension strategy; otherwise, if entering the brand extension market too early, the expenditures of marketing investment will not be recovered and be very hard for enterprise to create the sustainable competitive advantage.

Exhibit 6. Sensitivity analysis of growth option value $F_2(Y)$ to marketing investment of brand extension I_2

marketing investment of brand extension I_2	18	20	22	24	26
growth option value $F_2(Y)$	18.53	17.26	16.27	15.48	14.82

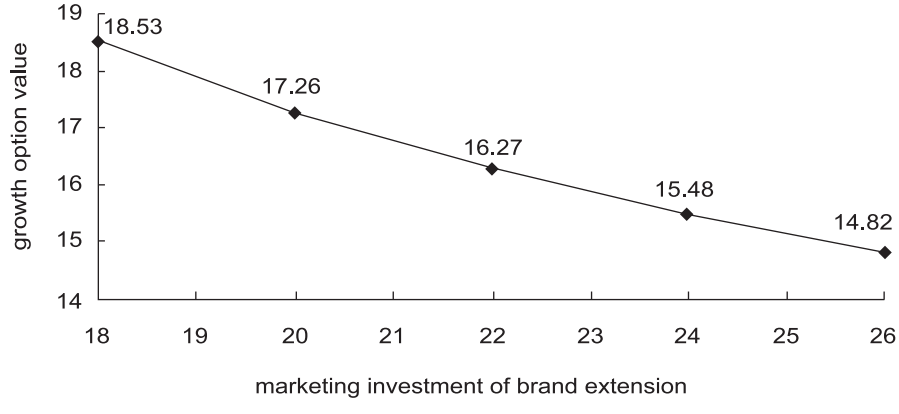

Figure 4. Sensitivity of growth option value to marketing investment for brand extension

D. The effect of marginal operation cost on growth option value of brand extension

When a firm starts to extend its core brand, it could make best use of its core brand equity, expand its sale amounts (Farquhar, 1990; Keller, 1993), decrease the failure probability of brand extension (Tauber, 1988), and decrease the costs of product introduction (Aaker & Keller, 1990). When it is willing to make use of its core competence in core brand, the level of brand satisfaction and loyalty is higher. Based on the successful experiences and learning at the first BIS stage (brand signaling), the marginal operation cost for brand extension could be easily controlled and decreased. As a result, the marketing expenditure for brand extension is lower than that for BIS stage ($w_m > w_s$) , and thus increases the growth option value. For example, among other things being equal ($\varepsilon_m = \varepsilon_s$=1.5) and under the return to scale, the marginal operation cost for brand extension decreases from 1.0 to 0.8, the growth option value, $F_2(Y)$, increases form 12.85 to 14.91 (see Exhibit 7 and Figure 5).

Exhibit 7 Sensitivity analysis of growth option value $F_2(Y)$ to marginal operation cost w_s under ($\varepsilon_m = \varepsilon_s$=1.5)

marginal operation cost w_s	0.8	0.9	1.0	1.1	1.2
growth option value $F_2(Y)$	14.91	13.78	12.85	12.06	11.38

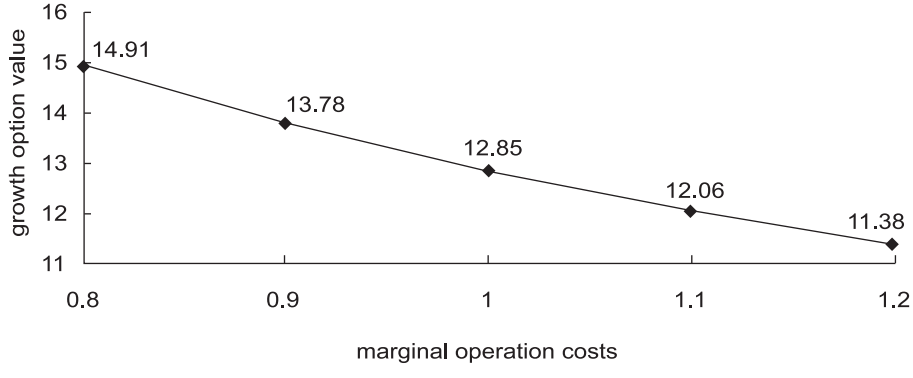

Figure 5. Sensitivity of growth option value to marginal operation cost

V. Conclusion and further research

A. Conclusion

The establishments of the BIS create the brand personality and provide the external clues of the product or service quality for customers (Aaker & Joachimsthaler, 2000). The recognition and the memory of the uniqueness of the products could enhance the

relationship between customers and the brand (Keller, 1997). Once the competitiveness of the brand has been built, the utilization of the brand extension and the scaled economy of the brand marketing could also reduce the risks of the customers' perceived risks (Dasgupta, 1988) through the trusts of the brand recognition and the related information (Janiszewski & Van Osselaer, 2000).

The previous researches adopt the NPV approach to analyze the impacts of the brand extension from the perspective of the customers, such as the consistency of the brand concepts (Park et al., 1991), the association of the brands for customers (Broniarczyk & Alba, 1994), the brand intensity (Reddy et al., 1994), and the marketing campaign of the brand extension (Hardie, 1994). But they all neglected the specific and irreversible investments of the BIS and the brand extension. Moreover, the traditional NPV, used to valuate a brand form the perspective of a firm, neglects the specific investments in different industrials and thus neglects the managerial flexibility value. The real options approach could amend such pitfalls (Dixit & Pindyck, 1994; Kogut & Kulatilaka, 1994; Brennan & Trigeorgis, 2000). The ROA considers the investment on brand extension as a sunk cost. Waiting for favorable conditions for brand extension is possible and valuable.

Unlike previous researches' focus on the valuation from the perspective of customers, this paper explores the brand extension value in terms of demand volatility and elasticity from the perspective of a firm. The irreversible sunk costs and marginal marketing expenditures are also integrated in the model to investigate their impacts on the value of brand extension. The decisions on the optimal entry time for brand extension to obtain the maximum option value are discussed in this paper.

The benefits of the brand extension are verified by many researchers, but its negative impacts of brand dilution and brand confusion should be noticed (Loken & John, 1993). Reddy et al. (1994), Kotler (1999), and Sullivan (1992) pointed out that the entry time of the brand extension has great impacts on its success, which could be accurately described by the real option approach. Dias and Ryals (2002) pointed out that companies should make better use of the brand platform to catch the value of the flexibility. Otherwise, the value of the brand equity will be underestimated. Even in the discrete time model, if a company only considers the single-stage of growth option, the true value of the option can not be correctly valuated. In view of this, this paper modifies the discrete single-stage model of brand extension valuation by Dias and Ryals (2002) and proposes a two-stage model: the BIS and the brand extension stages in the context of the continuous time, which extends those models proposed by Dixit and Pindyck (1994). Compared with the previous works, this

model considers the demand elasticity at brand extension stage, which is different from that at BIS stage. The different demand elasticity is caused by the effects of brand association or brand dilution, in the context of two-stage brand investment decisions on the continuous time basis. In the maximization of the total profits, the thresholds for both two stages are obtained to provide marketers with better brand investment decisions.

Compared with the previous studies on ROA, this work obtains the closed-form solution. Based on the comparative statics and sensitivity analyses, the following results are summarized:

(1) The higher fitness between core and extensive brands causes the positive associative benefits on the extensive brand. Thus the elasticity of demand for extensive brand will be smaller than that for core brand ($\varepsilon_m \geq \varepsilon_s$). The market power of the enterprise in the extensive market will be increased. It represents that brand extension can bring in the higher growth option value. On the contrary, when both the core and extensive brands do not fit well, the brand dilution will occur. Thus the demand elasticity of extensive brand will be higher than that of core brand ($\varepsilon_m \leq \varepsilon_s$) and the market power for the extensive market will be decreased. It implies that brand extension can not bring in the higher growth option value.

(2) When the market conditions are quite uncertain and volatile, implementing the brand extension rashly will cause the losses and the negative effects. Instead, waiting for the best investment opportunities derived from the market demand volatility (σ) and then choosing the optimal timing for brand extension will elaborate the strategic elasticity benefits of the brand extension.

(3) To implement the brand extension, the enterprise must devote the extra strategic marketing expenditures to the brand project. Too early entry to the market will cause the investment become irreversible. Therefore, when the growth of option value derived from the marketing investment expenditures (I_2) for brand extension excesses the investment threshold, the enterprise can undertake the brand extension strategy.

(4) Enterprises with more marketing experiences on the development of the core brand will benefit from the decreased marginal operational cost ($w_m > w_s$). That is, the efficient operations and cost reductions will increase the growth option values at brand extension stage.

B. Further Research

This work proposes an extended model with two-stage option: the BIS and the brand extension stages. The assumptions are made by this work that the market condition is less volatile and almost certain at the first BIS stage, while the market condition is more volatile at the second brand extension stage. At the second stage, the brand extension strategy is considered as a route to future growth. The further researches may release these assumptions to the more volatile market conditions for both the BIS and the brand extension stages. The resulting impacts can be compared with those in this work.

The model in this work discusses the brand extension strategy in s single company, and neglects the interactions among the brand competitors. It is suggested that the impacts of the first mover advantage (Leiberman & Montgomery, 1998) and the second mover advantage (Hoppe, 2000) can be explored in more details. Besides, the game theory (Smit & Trigeorgis, 2004) can also be used to explore the interactions among the brand competitors to find out the different value creation processes.

This work derives the decisions for the BIS and the brand extension by a sequential two-stage growth option in the context of the continuous time. For simplicity, the previous experiences of the existing brands are not considered. It is also suggested that the accumulated experiences of the previous successes and failures can be considered in terms of the learning curve (Majd & Pindyck, 1989). It may be quite interesting to explore the process to internalize the learning curve effects in the decisions for the BIS and the brand extension.

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Appendix

Appendix 1：Proof of Lemma 1

At time *t*, we create a portfolio with one unit of the brand extension project and n units of a short sale of segmented market uncertainty so that this portfolio is risk-free. Holding this portfolio in a very short interval of time $[t,t+dt]$, the investor obtains a yield, Ydt, over the period *dt*. However, a dividend $\delta \cdot Ydt$ is paid out so that the portfolio holder receives $(Y-n\delta Y)\,dt$. The brand extension creates the optimal profit flow, π^*. The capital gain from selling this portfolio is:

$\Pi = V - nY$,

Then

$$d\Pi = dV - ndY - \delta nYdt + \pi^* dt$$
$$= \left[\frac{1}{2}\sigma^2 Y^2 V''(Y) + (r-\delta)YV'(Y) - n(r-\delta)Y - n\delta Y + \pi^*\right]dt + (\sigma YV'(Y) - n\sigma Y)\,dz. \quad \text{(A1)}$$

If we set$n\ V'(Y)$, then the random term *dz* disappears. As a result, this portfolio is risk-free. Consequently, (A1) becomes:

$$\left(\frac{1}{2}\sigma^2 Y^2 V''(Y) - \delta YV'(Y) + \pi^*\right)dt. \quad \text{(A2)}$$

Let (A2) be equal to the risk-free return $r[V(Y) - nY]dt$. We hence have the differential equation in equation (12) as follows:

$$\frac{1}{2}\sigma^2 Y^2 V''(Y) + (r-\delta)YV'(Y) - rV(Y) + \pi^* = 0.$$

Appendix 2：Proof of Lemma 2

Assume that the solution to equation (12) takes the form:

$$V(Y) = A/r + (B \cdot Y^{\varepsilon_s})/\Delta. \quad \text{(B1)}$$

Solving for the following equation:

$$\frac{1}{2}\sigma^2 Y^2 V''(Y) + (r-\delta)YV'(Y) - rV(Y) + \pi^* = 0, \quad \text{(B2)}$$

where $V'(Y) = \varepsilon_s (B \cdot Y^{\varepsilon_s - 1})/\Delta$ ； $V''(Y) = \varepsilon_s(\varepsilon_s - 1)(B \cdot Y^{\varepsilon_s - 2})/\Delta$

$$\frac{1}{2}\sigma^2 Y^2 \varepsilon_s(\varepsilon_s - 1)(B \cdot Y^{\varepsilon_s - 2})/\Delta + (r-\delta)Y\varepsilon_s(B \cdot Y^{\varepsilon_s - 1})/\Delta - rV(Y) + A + B \cdot Y^{\varepsilon_s} = 0,$$

$$\frac{1}{2}\sigma^2\varepsilon_s(\varepsilon_s-1)(B\cdot Y^{\varepsilon_s})/\Delta+(r-\delta)\varepsilon_s(B\cdot Y^{\varepsilon_s})/\Delta-r(A/r+B\cdot Y^{\varepsilon_s}/\Delta)+A+B\cdot Y^{\varepsilon_s}=0,$$

$$\frac{\left(\frac{1}{2}\sigma^2\varepsilon_s(\varepsilon_s-1)+(r-\delta)\varepsilon_s-r\right)}{\Delta}+1=0,$$

We obtain

$$\Delta=r-(r-\delta-\frac{1}{2}\sigma^2)\varepsilon_s-\frac{1}{2}\sigma^2\varepsilon_s{}^2. \tag{B3}$$

Appendix 3：Proof of Proposition 1

According to Theorem 7.5 in Malliaris and Brock (1982), the option value of an investment project, denoted by F_2 (Y), can be derived by the method of dynamic programming and the Bellman equation is:

$$rF_2(Y)dt=E(dF_2(Y)) \quad . \tag{C1}$$

Equation (C1) says that the expected total return from an investment opportunity over a short period of time *dt* equals the expected growth of capital.

To solve for F_2 (Y) and the optimal threshold for demand shock, Y^*_2, at the second stage, we can derive (C2) , a differential equation, by the Ito's Lemma.

$$\frac{1}{2}\sigma^2Y^2F_2''(Y)+(r-\delta)YF_2'(Y)-rF_2(Y)=0. \tag{C2}$$

Equation (C2) are subject to the following boundary conditions:

$$F_2(0)=0, \tag{C3}$$

$$F_2(Y_2^*)=V(Y_2^*)-I_2, \tag{C4}$$

$$\text{and} \quad F_2'(Y_2^*)=V'(Y_2^*) \tag{C5}$$

Equation (C3) implies that if the extension value is zero, then its option value is also zero. Equation (C4) is the value matching condition, which indicates that the value of the extension option is equal to the net value of the brand extension at the cost of sunken expenditure, I_2 . Equation (C5) is the smooth-pasting condition, which suggests that at the point of optimal entry time, F_2 has to be continuous, smooth, and has the same slope as V (Y) does.

Suppose the solution of $F_2(Y)$ takes the following form (Dixit & Pindyck, 1994)

$$F_2(Y) = D_2 \cdot Y^{\beta_2}, \tag{C6}$$

Therefore, we have

$$F_2'(Y) = D_2 \cdot \beta_2 \cdot Y^{\beta_2 - 1} \quad \text{and} \quad F_2''(Y) = D_2 \cdot \beta_2 \cdot (\beta_2 - 1) Y^{\beta_2 - 2}. \tag{C7}$$

Substituting Equation (C3) into Equation (C2), we obtain:

$$D_2 Y^{\beta_2} \left[\frac{1}{2}\sigma^2 \beta_2^{\,2} + (r - \delta - \frac{1}{2}\sigma^2)\beta_2 - r \right] = 0, \tag{C8}$$

Since $D_2 Y^{\beta_2} \neq 0$, $\frac{1}{2}\sigma^2 \beta_2^{\,2} + (r - \delta - \frac{1}{2}\sigma^2)\beta_2 - r = 0$.

We can obtain $\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{(r-\delta)}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1.$ (C9)

Similarly, we substitute Equation (C6) into the boundary conditions, we have:

$$\begin{aligned} D_2 &= \frac{\varepsilon_s B}{\beta_2 \Delta} (Y_2^*)^{\varepsilon_s - \beta_2} \\ &= \frac{\varepsilon_s B}{\beta_2 \Delta} \left[\frac{(I_2 - A/r)\Delta\beta_2}{B(\beta_2 - \varepsilon_s)} \right] (Y_2^*)^{-\beta_2} \\ &= \frac{\varepsilon_s}{(\beta_2 - \varepsilon_s)} \cdot (I_2 - \frac{A}{r}) \cdot (Y_2^*)^{-\beta_2} > 0 \end{aligned}, \tag{C10}$$

and $Y_2^* = \left[\frac{(I_2 - A/r)\Delta_2\beta_2}{B(\beta_2 - \varepsilon_s)} \right]^{\frac{1}{\varepsilon_s}}$. (C11)

Appendix 4： Proof of Proposition 2

The option value for the BIS brand investment, $F_1(Y)$ has to satisfy the following differential equation:

$$\frac{1}{2}\sigma^2 F_1''(Y) Y^2 + (r - \delta) F_1'(Y) Y - r F_1(Y) = 0, \tag{D1}$$

It is subject to the following boundary conditions:

$$F_1(Y_1^*) = F_2(Y_1^*) - I_1, \tag{D2}$$

$$F_1'(Y_1^*) = F_2'(Y_1^*), \tag{D3}$$

and $\lim_{Y\to\infty} F_1(Y) < \infty$. (D4)

Substituting Equation (21) into Equation (D1), we obtain:

$$D_1 Y_1^{\beta_1}\left[\frac{1}{2}\sigma^2\beta_1^2 + (r-\delta-\frac{1}{2}\sigma^2)\beta_1 - r\right] = 0. \tag{D5}$$

Since $D_1 Y_1^{\beta_1} \neq 0$, $\frac{1}{2}\sigma^2\beta_1^2 + (r-\delta-\frac{1}{2}\sigma^2)\beta_1 - r = 0$.

We can obtain $\beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{(r-\delta)}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$. (D6)

Suppose the solution of $F_1(Y)$ takes the following form (Dixit & Pindyck, 1994)

$$F_1(Y) = D_1 \cdot Y^{\beta_1} \quad , \tag{D7}$$

Substituting the boundry conditions in Equations (D2)-(D4) into equation (D7), we obtain:

$$\frac{D_2\beta_2}{\beta_1} \cdot (Y_1^*)^{\beta_2-\beta_1} \tag{D8}$$

Finally, substituting Equation (D8) into Equation (D7), we can derive the demand shock threshold at the first stage as follows:

$$Y_1^* = \left[\frac{\beta_1 I_1}{(\beta_1-\beta_2)D_2}\right]^{\frac{1}{\beta_2}}. \tag{D9}$$

Appendix 5：Proof of Proposition 3

For a brief derivation, we use the following results by Dixit and Pindyck (1994),

$$\beta_2 = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 1, \tag{E1}$$

and

$$\Delta = r-(r-\delta-\frac{1}{2}\sigma^2)\varepsilon_s-\frac{1}{2}\sigma^2\varepsilon_s^{\,2}>0\,. \tag{E2}$$

Note that

$$Y_2^* = \left[\frac{(I_2 - A/r)\Delta\beta_2}{B(\beta_2-\varepsilon_s)}\right]^{\frac{1}{\varepsilon_s}}.$$

Differentiating it with respective to σ, we have

$$\frac{\partial Y_2^*}{\partial\sigma} = \left(Y_2^*\right)^{1/\varepsilon_s-1}\frac{I_2-\frac{A}{r}}{B(\beta_2-\varepsilon_s)^2}\left[-\beta_2(\beta_2-\varepsilon_s)(\varepsilon_s-1)\sigma-\Delta\frac{\partial\beta_2}{\partial\sigma}\right]. \tag{E3}$$

Substituting the partial derivative

$$\frac{\partial\beta_2}{\partial\sigma} = \frac{\beta_2}{\sigma}\left[-1+\frac{r-\delta+-\frac{1}{2}\sigma^2}{\sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2+2r\sigma^2}}\right]<0$$

into Equation (A2), it becomes

$$\frac{\partial Y_2^*}{\partial\sigma} = \frac{Y_2^*(\beta_2-1)\sigma}{(\beta_2-\varepsilon_s)\Delta_2}[\xi] \tag{E4}$$

where $\xi = \dfrac{\Delta}{\sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2+2r\sigma^2}}-\dfrac{(\beta_2-\varepsilon_s)(\varepsilon_s-1)}{(\beta_2-1)}.$ (E5)

We use the condition, $\beta_2 > \varepsilon_s$, provided by Dixit and Pindyck (1994) and the condition of the nonnegative equilibrium quantities, that is, $\varepsilon_s > 1$. Thus, we have $\beta_2 > \varepsilon_s > 1$. According to Equations (E2) and (E4) and the above condition, $\partial Y_2^* / \partial\sigma > 0$ if $\xi > 0$. Note that

$$\frac{\partial^2\xi}{\partial\varepsilon_s^2} = \psi^{-1}\sigma^2\left[\sigma^2+2(r-\delta+\sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2+2r\sigma^2})\right]>0 \tag{E6}$$

where $\psi = 2\sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2+2r\sigma^2}\cdot\left[\sqrt{(r-\delta-\frac{1}{2}\sigma^2)^2+2r\sigma^2}-(r-\delta+\frac{1}{2}\sigma^2)\right]>0\,.$

Hence, $\partial \xi / \partial \varepsilon_s$ is a strictly increasing function of ε_s. Moreover, since $\partial \xi / \partial \varepsilon_s \mid_{\varepsilon_s = \beta_2} = 0$ and $\partial \xi / \partial \varepsilon_s \mid_{\varepsilon_s = 1} = -4\delta\sigma^2 \Psi^{-1} < 0$, thus $\partial \xi / \partial \varepsilon_s < 0$ in the interval of $\beta_2 > \varepsilon_s > 1$. Therefore, ξ is also a monotone function of ε_s. In addition, since $\xi \mid_{\varepsilon_s = \beta_2} = 0$ and $\xi \mid_{\varepsilon_s = 1} = \delta [(r - \delta - \frac{1}{2}\sigma^2)^2 + 2r\sigma^2]^{\frac{1}{2}} > 0$, we have $\xi > 0$ in the interval of $\beta_2 > \varepsilon_s > 1$.

Consequently, we have

$$\partial Y_2^* / \partial \sigma > 0 . \tag{E7}$$

From **Proposition 1**, the option value of the brand extension is

$$F_2(Y) = D_2 \cdot Y^{\beta_2}, \tag{E8}$$

and

$$\frac{\partial F_2(Y)}{\partial \sigma} = \frac{\partial F_2(Y)}{\partial Y} \cdot \frac{\partial Y}{\partial \sigma}, \quad \frac{\partial F_2(Y)}{\partial Y} = D_2 \cdot \beta_2 \cdot Y^{\beta_2 - 1}, \quad D_2 > 0, \beta_2 > 1 .$$

We can finally obtain $\frac{\partial F_2(Y)}{\partial \sigma} > 0$. (E9)

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