BARRIERS TO INTERNATIONAL INVESTMENT, EXCHANGE RATE RISK AND WORLD CAPM

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ABSTRACT

Due to a mis-interpretation of mathematics, a vintage world CAPM that assumes barriers to international investment exist in the form of prohibitive taxes, is found to come up with wrong results. Security market lines for long and short international holdings are shown not to be parallel to that for holding domestic assets, as opposed to the claim by the original study. This result would have far-reaching empirical implications. In addition, based on recent empirical studies, the model is extended to incorporate forward hedging of exchange rate risk. The hedging might be a better approach than simply assuming away the risk.

Key Words: Hedging, Forward contract, Security market line, International investment.

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I. INTRODUCTION

In pursuit of reality, various authors have introduced barriers to international investment into the derivation of world CAPM[1]. Errunza and Losq (1985) assumes that a class of investors are unable to trade in a subset of securities as a result of portfolio inflow restrictions imposed by some governments. Their model results in two non-parallel Security market lines representing risk-return relationships for eligible and ineligible securities. Eun and Janakiramanan (1986) and Hietala (1989) discuss legal restrictions imposed by the government on the fraction of equities of local firms that can be held by foreigners. The unrestricted shares sell at a premium relative to the restricted shares. And yet, a general approach, notably adopted by Black (1974) and Stulz (1981), to modeling barriers to international investment that conserves the very essence of the above specific ones, is to impose a prohibitive tax on foreign asset holding.[2] Black assumes that an investor is required to pay taxes in proportion to the net holdings of risky foreign assets. His model renders a result, as pointed out by Stulz, that an increase in barriers to international investment would not induce domestic investors to abstain from holding foreign risky securities. If the barriers were very high, domestic investors would hold large amounts of foreign securities short.

Stulz argues that a barrier is a barrier. It would make it difficult to hold - either long or short - risky foreign assets. He proposes a model in which an investor pays a tax proportional to the absolute value of total holdings of risky foreign assets. The results are three parallel security market lines representing equilibrium risk-return relationship for holding foreign assets long, holding domestic assets, and holding foreign assets short respectively. In this paper, we show that the parallel stems from a mis-interpretation of first-order conditions of the optimization problem. Namely, Stulz interprets the first-order conditions as implying the existence of non-traded assets, while they actually imply that an investor would not hold a risky foreign asset long and short simultaneously. With the observation, we derive separate security market lines for long and short foreign holdings that appear not to be parallel to each other as claimed by Stulz. This result

1 All papers in the following, rely on CAPM concepts as some others who postulate the extremes of either complete segmentation or none at all. See Adler and Dumas (1983) for further references.

2 Recently, Bonser-Neal, Brauer, Neal and Wheatley (1990) discusses possible ways to empirically test all models mentioned above.

might havc important empirical implications that, for example, would question the universal validity of the hypothesis tested in Bonser-Neal, Brauer, Neal and Wheatley (1990).

Furthermore, a common assumption made by all afore-mentioned studies is the absence of exchange rate risk. As exchange rate risk could adversely affect the performance of international portfolios, investors would initiate hedging programs to shun the risk. In this paper, an effective approach to hedging that is based on the empirical studies of Adler and Simon (1986), and Eun and Resnick (1988), is incorporated into the derivation of the model. The hedging produces additional terms of forward exchange premium factor in various security market lines, since the approach advocates using foreign exchange forward contracts to hedge. Incidentally, these terms do not overrule the conclusion that security market lines are not parallel to each other.

The paper is organized as follows: Section I derives the correct security market lines from the original model setting. Section II makes the comparison of our results with that of stulz, and discusses some empirical implications of the differences. Finally, Section III concludes the paper.

II. THE CORRECT DERIVATION

A. The Original Model

The assumptions and notations below generally follow those in Stulz (1981), except for some minor changes. Assume

(a) there are only two countries, the domestic country D and the foreign country F,

(b) a safe real bond in each country that earns a common riskless rate r,

(c) neither transportation cost nor tariff exists,

(d) only domestic investors face barriers to international investment, foreign investors do not,

(e) a common utility function which depends positively on expected end-of-period wealth is maximized by typical investors.

Denote

k = an investor,

i = an asset,

t = the tax rate that represents barriers to international investment,[3] and

$\tilde{R}_i$ = the return on asset i for a foreign investor.

Therefore,

$\tilde{R}_i - t$ = the return when a domestic investor holds a foreign risky asset i long,

$-(\tilde{R}_i + t)$ = the return when a domestic investor holds i short and keeps the proceeds in cash.

Also denote

N = the number of total domestic and foreign risky assets,

n = the number of domestic risky assets,

$\bar{R}$ = a (N x 1) vector of expected returns on N risky assets,

$\bar{R}^k$ = the expected return of investor k,

$\underline{V}$ = a (N x N) matrix representing the variance-covariance of risky assets,

$\underline{L}^k$ = a (N x 1) vector of fractions of investor k's holding long in risky assets, all elements are nonnegative, he first n rows represent domestic risky asset holdings,

$\underline{S}^k$ = a (N x 1) vector of fractions of investor k's holding short in risky assets.

Therefore,

$\underline{L}^k - \underline{S}^k$ = the portfolio of investor k's risky assets holding, and

$(\underline{L}^k + \underline{S}^k)' \underline{e}\, t W^k$ = the taxes a domestic investor k has to pay,

where

W^k = investor k's total wealth,

e = a (N x 1) vector which has zeros in its first n rows and ones everywhere.

A typical investor k's problem, as modeled by Stulz is to:

[3] Please refer to Black (1974) and Stulz (1981) for discussions on this type of barrier.

$$\text{Min } 1/2\,(L^k - S^k)'\,V\,((L^k - S^k)$$

subject to:

$$(L^k - S^k)'\bar{R} - (L^k + S^k)'et + [1 - (L^k - S^k)'\alpha]\,r \geq \bar{R}^k, \tag{1}$$

$$L^k \geq 0, \tag{2}$$

$$S^k \geq 0, \tag{3}$$

where
α = a (N × 1) vector of ones ,
0 = a (N × 1) vector of zeros .

B. The Extension to Incorporate Forward Hedging

The model can be improved in the sense of mean-variance portfolio efficiency.As is well-understood, international investments involve exchange rate risk and that exchange rate uncertainty is a largely nondiversifiable factor adversely affecting the performance of international portfolios. It would be desirable for an international investor to effectively control exchange rate volatility. However, it is notable that all studies of world CAPM with barriers mentioned earlier assume away exchange rate risk.

An arguably preferable way to deal with the risk in a theoretical setting is to make use of the empirical results of Eun and Resnick (1988). Based on the work of Adler and Simon (1986) which asserts that a foreign stock market investment has approximately unitary exposure exclusively to its own currency and to no other, Eun and Resnick confirm that an effective way to control exchange rate risk is to use foreign exchange forward contracts on one-for-one basis. That is, an international investor would sell the expected foreign currency proceeds forward.[4] Eun and Resnick empirically show that portfolio selections with forward hedging outperform unhedged portfolios. Therefore, if a typical investor is assumed to be rational with respect to mean-variance efficiency, he or

[4] We ignore the problem of estimation risk here. For a discussion on how to deal with this problem, please refer to Eun and Resnick (1988).

she would be willing to use foreign exchange forward contracts to hedge exchange rate risk. After the initiation of forward hedging, the investor could essentially act as if there were no risk associated with changes in relative currency values.

Thus assumed, the hedging can be achieved by substituting the vector, (R + f), for the vector of the expected rate of return, R, in the constraint (1) of the optimization problem, where f is a (N X 1) vector of forward exchange premium with the first n rows being zeros.[5]

Straightforward as it may look, one should note that a common practice of hedging is to treat forward contracts as additional assets and incorporate them into portfolio selection - the so-called "portfolio approach to hedging" in the literature. In this approach, the numbers of forward contracts needed for effective hedging will be determined by the variance-covariance matrix of all risky assets, including forward contracts. This may not be a cost- and time-efficient way of hedging. The empirical results of Adler and Simon, and that of Eun and Resnick are encouraging as they show that simple one-for-one hedging is an effective approach to hedging exchange rate risk.

With forward hedging, the Lagrangian function for the extended optimization problem would be

$$
\begin{aligned}
£^k = & \left[-1/2\,(\underline{L}^k-\underline{S}^k)'\,\underline{V}\,(\underline{L}^k-\underline{S}^k)\right] \\
& +\Phi_k\{(\underline{L}^k-\underline{S}^k)'(\overline{\underline{R}}+\underline{f})-(\underline{L}^k+\underline{S}^k)'\underline{e}\,t \\
& +[1-(\underline{L}^k-\underline{S}^k)'\underline{\alpha}]\,r-\overline{R}^k\}
\end{aligned}
$$

where Φ_k is the Lagrange multiplier.

The Kuhn-Tucker conditions then, are

$$
\frac{\delta £^k}{\delta L^k}=\underline{V}(\underline{L}^k-\underline{S}^k)-\Phi_k(\overline{\underline{R}}+\underline{f}-r\,\underline{\alpha}-t\underline{e})\geq 0, \tag{4}
$$

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[5] The domestic-currency rate of return under the hedging strategy is:

$$
\tilde{R}^H_i=[1+E(R_i)](1+f_i)+[\tilde{R}_i-E(R_i)](1+\tilde{a}_i)-1
$$

where $\tilde{a}_i$ is the rate of appreciation of the foreign against the domestic currency. $\tilde{R}^H_i$ can be approximated by $(\tilde{R}_i+f_i)$. (see Eun and Resnick (1988) for details.)

$$\frac{\delta £^k}{\delta \underline{S}^k} = -\underline{V}(\underline{L}^k - \underline{S}^k) + \Phi_k(\overline{\underline{R}} + \underline{f} - r\underline{\alpha} + t\underline{e}) \geq 0, \tag{5}$$

$$(\underline{L}^k)' \frac{\delta £^k}{\delta \underline{L}^k} = 0, \tag{6}$$

$$(\underline{S}^k)' \frac{\delta £^k}{\delta \underline{S}^k} = 0, \tag{7}$$

$$\frac{\delta £^k}{\delta \Phi_k} = (\underline{L}^k - \underline{S}^k)'(\overline{\underline{R}} + \underline{f}) - (\underline{L}^k + \underline{S}^k)'\underline{e}t + [1 - (\underline{L}^k - \underline{S}^k)'\underline{\alpha}]r - \overline{R}^k \geq 0, \tag{8}$$

$$\Phi_k\left[\frac{\delta £^k}{\delta \Phi_k}\right] = 0, \tag{9}$$

$$\underline{L}^k \geq 0, \tag{10}$$

$$\underline{S}^k \geq 0, \tag{11}$$

$$\Phi^k \geq 0, \tag{12}$$

If we denote Vi as the ith row of V, then (4) and (5) can be combined to yield

$$\Phi_k(\overline{R}_i + f_i - r + t) \geq V_i(L^k - S^k) \geq \Phi_k(\overline{R}_i + f_i - r - t), \tag{13}$$

where

$V_i(L^k - S^k)$ = the covariance between the return on asset i and the return on the investor k's portfolio of risky assets.

It is from a similar inequalities (without terms involving forward exchange premiums) that Stulz defines a "non-traded" asset [6] as an asset for which both inequalities in (13) hold strictly. (Since, by definition, if the investor does not hold that asset, $L_i = S_i = 0$.) And after arguing for the possibility of its existence, he claims that only empirical research can answer the question of whether or not some assets would be non-traded for some investors.

However, it is easy to see that from the model setting, mathematically, when both inequalities in (13) hold strictly, it merely means that an investor is not allowed to hold an asset long and short simultaneously.[7] This is true if we look again into the definitions of returns on holding risky assets long and short. When an asset is held long and short simultaneously, the expected return on such a portfolio would always be (-2t), since the investor has to pay tax t for both long and short positions held.

The model is set up correctly to exclude the possibility that an investor would hold a risky foreign asset long and short simultaneously. Since the returns on holding the same asset long and short cancel each other, in the real world, special reasons are cited for investors to do so. For example,[8] a short selling can be prudently initiated to protect a profit in a stock at a time when the buyer does not want to sell it and take profit. If the stock is sold now, the buyer may have to pay a higher tax on that profit when he or she currently falls into a higher tax bracket. One alternative for the buyer is to hold on for another three months until the profit or capital gain can be reported as income in another tax year when a lower tax bracket is expected. For fear of decline in the stock value during that three months, a short selling initiated would offer the investor the insurance needed. When the investor is both long and short the same number of shares of the same stock, his position is stabilized. This is a case of tax-related hedging.

A second example can be found in a speculator's trading who has been pessimistic about the long term market trend and has sold short. When sensing a temporary market rebound, the short seller might initiate long positions in the same stocks in order to be relieved of the liquidity stress of margin calls. This is a case of pure hedging. Since the purpose here is to derive an equilibrium model based on a micro-agent's optimization problem, it is correct to drop special cases such as the ones mentioned above. Holding a risky asset simultaneously long and short is normally related to short-term, temporary

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[6] Non-traded assets are non-traded between countries.

[7] Stulz (1981, p. 927) states that "From first order conditions, it follows that both inequalities can hold strictly only if the investor does not hold that asset, ... "

[8] This example is from Fischer and Jordan (1979).

portfolio decisions that may not have profound market implications. Therefore, a non-traded asset that exists because both inequalities in (13) may prevail simultaneously, seems to be a mathematical artifact.[9]

We now derive security market lines separately for long and short positions. Denote L_i^k and S_i^k as the ith asset held by investor k long and short respectively, if $L_i^k > 0$, i.e., a long position is held, then from (4) and (6), we have

$$\underline{V}_i(\underline{L}^k - \underline{S}^k) = \Phi_k(\overline{R}_i + f_i - r - t). \tag{14}$$

Now, denote

$$T_k = \Phi_k W^k,$$

and

$$T_D = \Sigma_{k \in D} T_k,$$

$$T_F = \Sigma_{k \in F} T_k.$$

Then,

$$T_D + T_F = \Sigma_{k \in W} \Phi_k W^k,$$

where w represents the whole world (Two countries.)

Also denote

$$\tau_D = \frac{T_D}{T_D + T_F},$$

Ω = a (N x 1) vector whose element Ω_i is equal to the fraction of world wealth W_W supplied in the form of risky asset i.

[9] As pointed out by Errunza and Losq (1985), Stulz failed to specify the risk-return tradeoff for non-traded assets. After we re-derive the model, there will be no non-traded assets implied by the model. Then, the model becomes complete in the sense that no risk-return tradeoff for any assets is left out.

Assume all markets for risky securities are in equilibrium, from (14) we obtain

$$\Sigma_{k\in W}\,\underline{V}\,(\underline{L}^{k}-\underline{S}^{k})=\Sigma_{k\in W}\,\Phi_{k}\,W^{k}\,(\underline{\overline{R}}+\underline{f}-r\,\underline{\alpha}-t\,\underline{e})$$
$$=(T_{D}+T_{F})\,(\overline{R}+f-r\,\alpha-t\,e)$$
$$=(T_{D}+T_{F})\,(\underline{\overline{R}}+\underline{f}-r\,\underline{\alpha}-\tau_{D}\,t\,\underline{e})$$
$$(\text{ since } t=0 \text{ for } k\in F.)$$

Therefore,

$$\underline{V}\underline{\Omega}W^{W}\,(T_{D}+T_{F})\,(\underline{\overline{R}}+\underline{f}-r\,\underline{\alpha}-\tau_{D}\,t\,\underline{e}), \tag{15}$$

where W^{W} = the wealth of the whole world.

Premultiply (15) by $\underline{\Omega}'$ yields:

$$\underline{\Omega}'\,\underline{V}\,\underline{\Omega}\,W^{W}=\sigma_{W}^{2}\,W^{W}=(T_{D}+T_{F})\,(\underline{\Omega}'\underline{\overline{R}}+\underline{\Omega}'\underline{f}-r\,\underline{\Omega}'\underline{\alpha}-\underline{\Omega}'\underline{e}\,\tau_{D}\,t)$$
$$=(T_{D}+T_{F})\,(\overline{R}_{m}+\underline{\Omega}'\underline{f}-r-t_{m}), \tag{16}$$

where $t_{m}=\underline{\Omega}'\,\underline{e}\,\tau_{D}\,t$.

Dividing (15) by (16) yields

$$\frac{\underline{V}_{W}}{\sigma_{W}{}^{2}}=\underline{\beta}_{m}=\frac{\underline{\overline{R}}+\underline{f}-r\,\underline{\alpha}-\tau_{D}\,t\,\underline{e}}{\overline{R}_{m}+\underline{\Omega}'\underline{f}-r-t_{m}}.$$

Rearranging, we have the valuation model for asset i held long

$$\beta_{mi}\,[\,\overline{R}_{m}+\underline{\Omega}'\underline{f}-r-t_{m}\,]+\tau_{D}\,t=\overline{R}_{i}+f_{i}-r,\quad i\in F, \tag{17}$$

where

β_{mi} = the beta of common stock i computed using the world market portfolio.

Similarly, if asset i is held short, $S_{i}^{k}>0$, then from (5) and (7) yield

$$\underline{V}_i(\underline{L}^k - \underline{S}^k) = \Phi_k(\overline{R}_i + f_i - r + t).$$

Following the same procedures as above, we have for i held short

$$\beta_{mi}[\overline{R}_m + \underline{\Omega}'\underline{f} - r + t_m] - \tau_D t = \overline{R}_i + f_i - r, \quad i \in F, \tag{18}$$

Equations (17) and (18) can be combined to write the valuation in a compact way

$$\beta_{mi}[\overline{R}_m + \underline{\Omega}'\underline{f} - r - ht_m] + h\tau_D t = \overline{R}_i + f_i - r, \quad i \in F, \tag{19}$$

where
h = 1, if i is held long,
h = -1, if i is held short.

Without barriers and forward hedging, equation (19) degenerates to the Sharpe-Lintner Model for domestic assets. The two security market lines are apparently not parallel to each other. Also, if forward hedging is not initiated, the absence of terms involving forward exchange premium will not overrule the non-parallel of the security market lines.

III. COMPARISON OF OUR RESULTS WITH STULZ'S

Without the extensions made in this paper, Stulz derives the security market line for holding foreign asset long

$$\beta_{mi}[\overline{R}_m - r - t_m + q_m] + \tau_D t = \overline{R}_i - r, \quad i \in F, \tag{20}$$

as well as the security market line for holding foreign asset short

$$\beta_{mi}[\overline{R}_m - r - t_m + q_m] - \tau_D t = \overline{R}_i - r, \quad i \in F, \tag{21}$$

where

$q_m = \underline{\Omega}' q_D \tau_D$,
$q_D = \Sigma_{k \in D} \pi^k q^k$,
$\pi^k = T^k / T_D$, and
q^k = slack variables that make the inequality of (4) into equality.

The two lines are parallel. All risky foreign assets held long lie on a security market line which is above and parallel to the security market line of all risky foreign assets held short, as well as the line for holding domestic assets.

Comparing (20) and (21) with our (17) and (18) and ignoring the forward premium terms, a distinctive difference is the additional term q_m in both (20) and (21), and the opposite sign of t_m in (21). In other words, in (20), q_m should be zero and in (21), q_m should be $2t_m$. The differences come from the reasons discussed in the previous section. Namely, Stulz mis-interprets (13) as implying the existence of non-traded assets rather than reflecting the market observation that an investor would not hold an asset long and short simultaneously. As a result, in his derivation of (21), he ignores the uses of condition (6) and (7) in regard to q^k_i and Q^k_i, where Q^k_i is the slack variable that makes the inequality of (5) into equality.[10] This can be easily shown as, when a long position is initiated, by using (6), q^k_i would be zero and there would not be q_m in (20). Also, in (21), by definition,

$$\begin{aligned} q_m &= \underline{\Omega}' \underline{q}_D \tau_D \\ &= \tau_D \underline{\Omega}' \Sigma_{k \in D} \pi^k \underline{q}^k \\ &= 2 \tau_D t \underline{\Omega}' \underline{e} \\ &= 2 t_m . \end{aligned}$$

The third equality comes from the fact that $q^k_i + Q^k_i = 2t$, $i \in F$, Q^k_i Then, when a short position is initiated, by using (7), Q^k_i is zero, and so $q^k_i = 2t$.

Recently, based on empirical implications of studies of Black, Stulz, Eun and Janakiramanan, and Hietala, among others, Bonser- Neal, Brauer, Neal and Wheatley (1990) examines whether announcements of changes in investment restrictions affect

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10 When Stulz defines a non-traded asset as $L^k_i = S^k_i = 0$, conditions (6) and (7) are indeed used. But they are not used later in regard to q^k_i and Q^k_i.

changes in the premiums and discounts on closed-end country funds. Since closed-end country funds represent long positions taken indirectly by domestic investors in foreign assets, if a country's restrictions are binding, an announcement that they are to be tightened (loosened) should raise (lower) the premium on the country's fund. This conjecture is based on the parallel of security market lines for holding risky foreign assets and domestic assets. From our equation (19), since $t_m > 0$, the slope of the security market line for holding foreign asset long (short) will be smaller (larger) than that of holding domestic assets. In fact, they may cross each other. When this occurs, one can no longer claim unmistakenly that a tax increase will induce a raise in the premium on the country's fund. Perhaps the claim is true only for lower beta portfolios. This hypothesis calls for future empirical studies to verify.

IV. CONCLUSION

From our understanding of investors behavior, they hold the same risky foreign assets simultaneously long and short only for special reasons, such as hedging. In an equilibrium model, it is appropriate to exclude temporary special cases as they may not have profound market implications. Following this argument, it seems that the definition of non-traded assets in Stulz's study is at best a mathematical artifact, and that the security market lines for long and short positions in risky foreign assets are not parallel to each other as Stulz claimed. The result implies that the empirical conjecture in Bonser-Neal, Brauer, Neal and Wheatley's study may not be universally valid. Still, if one studies the intertemporal behavior of asset prices, the short-term hedging that leads to temporary holding a risky asset long and short simultaneously should not be overlooked, since the behavior may cause white noises in the stochastic process of equilibrium asset prices.

As for hedging exchange rate risk, this paper makes use of the results of recent empirical studies. However, there is no theoretical proof that the hedging technique is necessarily optimal. More empirical studies are in need to determine better and cheaper ways of hedging in international investments. Ideally though, the hedging will not cause unduly technical complications when one attempts to derive an equilibrium model.

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