Alternative Valuation of the Cost of Deposit Insurance: An Application of Option Pricing Model With Stochastic Volatility

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Abstract

This paper first introduces the theoretical application of Option Pricing Model with stochastic volatility to the valuation of deposit insurance premium. We compare our model with Merton's (1977) original version in which volatility is assumed as constant and show that Merton's model is just a special case of our generalized pricing model. Since Black-Scholes model frequently undervalues deep in- and out-of-the-money option, we expect that future empirical studies using this pricing model with stochastic volatility could provide higher estimates of deposit insurance premiums and resolve the controversy between the previous empirical finding of overcharged premiums and the insolvent situation of deposit insurance funds.

Key Words: Deposit Insurance Premium, Black-Scholes Option Pricing Model, Stochastic Volatility, Stochastic Differential Equation, Diffusion Process.

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I. INTRODUCTION

One major issue of the current research on deposit insurance concerns the desirability and feasibility of a risk-based deposit insurance system. Under such a system, the deposit insurance assessment would be related to the degree of risk that an insured institution poses to the deposit insurance funds. Alternatively, a flat-rate deposit insurance system which all insured institutions are assessed at the same rate for their deposit coverage and premiums are invariant to the level of risk that a bank poses to the insurance funds. The flat-rate premium structure has been often criticized for subsiding high-risk banks by penalizing more-conservatively-run banks and providing incentives for increased risk-taking.

On the research of risk-based deposit insurance premium, the use of the option-pricing approach to estimate appropriate premium rates are particularly favored by researchers because this approach offers at least two advantages relative to the use of historical, system-wide loss experience (see Scott and Mayer (1971), Humphrey (1976), Barth et al (1990)) First, contingent-claim analysis allows for bank-specific estimates of the correct premium. Second, the appropriate premium can be computed using data collected over fairly short time periods. After Merton (1977) first derived the option-pricing formula for the deposit-insurance premium , Marcus and Shaked (1984), Ronn and Verma (1986), Pennacchi (1987) and Miles and Kim (1988) have done empirical research applying the formula to the valuation of the U.S. deposit insurance. However, most cases of these research have found that U.S banks were over- charged by the deposit insurers. The over-charged findings are extremely controversial comparing with the financial condition of SAIF (Savings Association Insurance Fund) and BIF (Bank Insurance Fund) insurance funds. Both funds suffered huge insurance losses in the 1980s and 1990s. The FSLIC (Federal Savings and Loan Insurance Company) exhausted all its reserves and was declared insolvency by U.S. GAO in 1986 and even BIF was probably insolvent at the end of 1991 in GAO's best judgement[1]. The overcharged finding also contracdict the undercharged proposition modelled and argued by the Buser, Chen and Kane (1981).

The controversy and the contradiction inspired us to revise Merton's (1977) option- pricing formula and to derive a more general formula which might resolve the controversy. We focus our extension on the constant volatility assumption of Merton (1977) for the following four reasons. First, it is well known that the volatility of stock price plays a central role in option-pricing model. The significantly positive relation between the deposit insurance premium and asset volatility are also supported by Merton's (1977) simulation estimates as we plotted out in the Figure 1. Second, Pyle(1983) and Marcus and Shaked (1984) pointed out that small errors in the estimation of the volatility could have major effects on the value of deposit insurance premium. Third, the recent option literature (Johnson and Shanno (1987), Scott(1987), Wiggins (1987)) suggest the constant volatility version of Black-Scholes option model frequently undervalues deep in- and out-of-the-money options. Fourth, as documented in Hull and White (1987), the stock price volatility follows diffusion process in reality.

With these observations in mind, we started out with Merton's (1977) model and derive the valuation formula of deposit insurance by applying an option-based model with stochastic volatility.

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[1] See GAO/AFMD-91-90.

II. THE OPTION-BASED MODEL WITH STOCHASTIC VOLATILITY

As it is well-known that the Black-Scholes techniques can be applied to the pricing of corporate liabilities. Merton (1977) applied their original analysis and presented a systematic theory for determining the cost of deposit insurance. The foundation for such pricing model of the deposit insurance is the isomorphic relationship between deposit insurance contract and common stock put option[2].
Based on this isomorphic relationship, we analytically derive the option-pricing formula for the value of deposit insurance with stochastic volatility by adopting Hull and White's (1987) approach. Hull and White (1987) successfully derived a pricing formula for a call option with stochastic volatility by assuming that the volatility is uncorrelated with the stock price. In this section, we are going to apply their model to the valuation of deposit insurance based on the isomorphic correspondence between stock call option and bank equity, as well as between stock put option and deposit insurance contract. In this valuation model, we are not only look at the case when the volatility of bank assets is uncorrelated with the market value of bank's assets, but also examine the case when the volatility is correlated with the market value of bank's assets.

Before formally deriving the valuation model for bank equity under two different assumptions about the stochastic volatility, we summarize the assumptions made and the notations used in this model.

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2 However, in a diffrent perspecitve, Kane(1986) and Kaufman(1992) argued that the FDIC frequently does not accept delivery of the bank as soon as the strike price is reached but instead often fornears the decapitlized bank.

A. Notations and Assumptions of the Model

Notations:

A: the market value of bank assets.

B: the total debt of the bank at the time of next auditing.

D: the aggregate deposits of the bank at current time.

g: a known constant percentage growth rate in deposits.

R: the interest rate on banks deposits.

s: the interest rate paid in the form of services.

α : the instantaneous expected rate of return on the bank assets per unit time.

V: the instantaneous variance of the return on the bank assets per unit time ($V = \sigma^2$)

$\bar{V}$: the mean variance of bank assets return over the period between two consecutiveauditing time.

μ_A : the drift parameter of A depending on A, V, and t.

μ_V : the drift parameter of V depending on V and t.

α : the drift parameter of V depending on V and t.

ξ : the parameter that may depend on V and t.

E : the market value of bank equity where ET = Max(V_T-B, 0) and Et = E(A_t, V_t, T-t; B).

ρ : the correlation coefficient between the stochastic volatility and the market value ofban k's assets.

dz_A, dz_V : standard Gauss-Wiener processes.

dw_A, dw_V : standard Gauss-Wiener Processes.

G(A_t, V_t, T-t; B): the bank's deposit insurance value.

Assumptions of the Basic Model

(1) The dynamics for the value of a given bank assets follow the diffusion-type stochastic process with stochastic differential equa-

tion :

$$dA = [\,\alpha - (R+s)D]\,dt + dD + \sigma A\,dz_A, \quad \text{or}$$

$$dA = [\,\alpha A - (R+s\text{-}g)D]\,dt + \sigma A\,dz_A. \tag{1}$$

Let $\mu_A = \alpha - (R+s\text{-}g)\,(D/A)$, then we can rewrite the diffusion process for the bankassets as follows:

$$dA = \mu_A A\,dt + V^{1/2} A\,dz_A \tag{2}$$

where $\mu_A = \mu_A(A, V, t)$ and $V^{1/2} = \sigma$.

(2) The variance of the bank's assets' return is stochastic and follows the stochastic differential equation as

$$dV = \mu_V V\,dt + \quad V\,dz_V \tag{3}$$

where $\mu_V = \mu_V(V, t)$, $\xi = \xi\,(V,t)$ and both do not depend on A.

We also assume that the Gauss-Wiener processes dz_A and dz_V are correlated with the correlation coefficient ρ , i.e.,

$$E(\,dz_A\,dz_V\,) = \rho\,dt \tag{4}$$

(3) The dynamics for bank's aggregate deposits D is non-stochastic and can be described by $dD/dt=gD$. So, the amount of deposits at the next auditing time is $B= (1+g)D$.

(4) The market for the banking industry is competitive without entry barriers.

(5) The deposit insurer charges the bank a one-time premium to insure all the deposits.

(6) The deposit insurer has suffieiecnt financial capability to absorb all possible losses and will meet its obligations.

(7) Trading in securities takes place continuously in time.

(8) The securities in exchange markets are "sufficiently perfect" and asset-return dynamics follows continuous-time version of Capital Asset Pricing Model.

(9) The agents in the economy can borrow or lend at the same risk-free rate denoted by r, which must be constant or at least deterministic.

(10) The total deposits comprise of bank's total debt.

(11) There is no transaction costs and no surveillance or auditing costs in this model.

(12) Both the regulator and the Banks are risk-neutral.

(13) The volatility has zero systematic risk and hence is uncorrelated with the aggregate consumption .

B. The Evaluation of Bank Equity and the Cost of Deposit Insurance Premiums

Following Merton's (1977) approach, we consider the simple model of a bank that raises money by issuing deposits. We assume that there are no interim interest payments required on the deposits, and so the deposits are term discount issues. If the bank does not have deposit insurance, the value of the bank's equity on the maturity date will be Max [0, A-B] where A represents the market value of the bank assets and B represents the value of the deposits on that date. If the bank buy the insurance for its deposits from the deposit insurer, then the value of this deposit insurance from the bank viewpoint is precisely a put option. The length of time until the next audit of the bank's assets is considered as the time to maturity of this put option and the value of the deposits on the maturity date is considered as the put option's exercise price. That is, when the market value of bank's assets falls below the bank's deposit at the next auditing time, then the guarantor must pay off the deposits by the amount of $(B\text{-}V_T)$. Hence, we can value the deposit insurance guarantee as a put option, i.e.,

$$GT(.,.,.,.) = \text{Max } (0,\ B\text{-}A_T)$$

where Gt (.,.,.) depends on the market value of bank's assets (A_t), the exercise price (B), the volatility of bank's assets (V_t), and the time length until the next auditing time.

Based on the assumptions and the above analysis, we can see that the only state variables affecting the bank equity value (E) is bank assets (A) and the volatility (V). Hence, E must satisfy the following differential equation[3]

$$(\partial E/\partial t)+(1/2)[\sigma^2A^2(\partial^2E/\partial EA^2)+2\rho\ \sigma^3\xi\ A(\partial^2E/\partial A\ \partial V) + \xi^2V^2(\partial^2E/\partial V^2)\text{-}rE = \text{-}rA\ (\partial E/\partial A) - \mu_V\ \sigma^2(\partial E/\partial V) \quad (5)$$

In order to solve (5) for the bank equity value E, we can apply the risk-neutral valuation method. When the volatility is uncorrelated with aggregate consumption, as we assumed here, the volatility has zero systematic risk and the bank equity value will not depend on risk preference. We confine our model in a risk-neutral world so that the bank equity value is just the present value of the expected terminal residual value of bank equity discounted at the risk-free rate. Hence, we can write the bank equity value as

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3 Any security f with a price that depends on state variable θ i must satisfy the differential equation

$$(\partial f/\partial t)+(1/2)\Sigma\ ij\ \ \sigma\ ij\ \rho\ i\ \rho\ j(\partial 2f/\partial\theta\ i\ \partial\theta\ j)\text{-}rf = \Sigma_i(\theta_i)[(\partial f/\partial\theta_i)[-\mu_i+\beta_i(\mu^*\text{-}r)]$$

(see Hull & White[1987]).

$$E(A_t, V_t, t) = e^{-r(T-t)} \int E(A_T, V_T, T)\ \phi(A_T|A_t, V_t)dA_T \tag{6}$$

where $\phi(A_T|A_t, V_t)$ is the conditional distribution of A_T given the current market value of bank's assets A_t and variance V_t at time t[4]. Under the risk neutral world, at any times, the bank assets value A_s will be given by $E(A_s|A_t) = A_t e^{r(s-t)}$. Let $\bar{V}$ represents the mean volatility over the two consecutive auditing time periods which can be written as

$$\bar{V} = [1/(T-t)] \int^T \sigma_s^2\ ds \tag{7}$$

Then, using the property of conditional density function, the conditional probability $\phi(A_T|A_t, V_t)$ can be denoted by

$$\phi(A_T|\sigma_t^2, A_t) = \int h(A_t|\bar{V})g(\bar{V}\ |\ \sigma_t^2, A_t)d\bar{V} \tag{8}$$

or

$$\phi(A_T|\sigma_t^2) = \int h(A_t|\bar{V})g(\bar{V}\ |\ \sigma_t^2, A_t)d\bar{V} \tag{9}$$

where h(.) and g(.) are conditional probability density functions[5]. If we plug (8) or (9) into (6), then we can write the current value of bank equity as either

$$E(A_t, V_t, t) = e^{-r(T-t)} \int\int E(A_T)h(A_T|\bar{V})g(\bar{V}\ |\ \sigma_t^2)dA_T\ \ d\bar{V} \tag{10}$$

$$\text{or } E(A_t, V_t, t) = \int \{e^{-r(T-t)} \int E(A_T)h(A_T|\bar{V})dA_T\}g(\bar{V}\ |\ \sigma_t^2)d\bar{V} \tag{11}$$

Now, we examine the property of equation (11) by considering the correlation coefficient between the volatility and the market value of bank's assets ρ in several special cases.

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4 Equation (6) is comparable to equation (6) in Hull and White (1987).

5 The At in J(9) is suppressed to simplify the notation.

(A) When the Volatility is Uncorrelated with the Market Value of Bank Assets (ρ =0)

When the volatility is uncorrelated with the bank equity value, the bank equity value E should satisfy the following stochastic differential equation

$$(\partial E/\partial t) + (1/2)[\sigma^2 A^2(\partial^2 E/\partial A^2) + \xi^2 V^2(\partial^2 E/\partial V^2) - rE = -rA(\partial E/\partial A) - \mu_V \sigma^2(\partial E/\partial V) \qquad (12)$$

Also, under the risk-neutral world, the market value of bank assets and its instantaneous variance V will follow the stochastic processes

$$dA = r\,A\,dt + \sigma\,A\,dw_A$$
$$dV = \alpha\,V\,dt + \xi\,V\,dw_V.$$

And the distribution of $\{A_T/A_0\}$ conditional on $\bar{V}$ is lognormal with mean $\exp^{rT-(\bar{V}T/2)}$ and variance exp $\bar{V}$ T. (Note that the distribution of log $\{A_T/A_0\}$ is not normal). The reason behind this is as follows. Since the parameters of the lognormal distribution depend only on the initial bank assets value, the risk-free rate, and the mean variance over the insurance contract period, any path that the variance V may follow and that has the same value of mean variance will produce the same lognormal distribution. (see Hull and White, 1987). This is also true even the variance is stochastic. Although there are an infinite number of paths that give the same mean variance,all of these paths still produce the same terminal distribution of bank assets value. Hence, the terminal distribution of the bank assets value given the mean variance is lognormal no matter whether the σ^2 is deterministic or stochastic.

Hence, the term inside the bracket in equation (11)

$$E(A_t,V_t,t) = \int [e^{-r(T-t)} \int E(A_T)\ h(A_T, V_T | \bar{V})dA_T]g(\bar{V} \mid \sigma^2)d\bar{V}$$

is the Black-Scholes price for the bank equity value when the bank assets has a mean variance $\bar{V}$. Hence. it can be represented by C(V) as follows:

$$C(\bar{V}) = A_t N(d1) - Be^{-r(T-t)} N(d2) \qquad (13)$$

where $d_1 = \log(A_t/B) + (r + (\bar{V}/2))\ (T-t)$
and $d_2 = \log(A_t/B) + (r - (\bar{V}/2))\ (T-t)$.

(Recall that B=(1+g)D is the exercise price in this model)
So, the value of bank equity is given by the following equation

$$E(A_t, V_t, t) = \int C(\bar{V})\ g(\bar{V} \mid \sigma^2)\ d\bar{V} \qquad (14)$$

the pricing equation of (14) is always true in a risk-neutral world when the volatility and bank equity value are instantaneous uncorrelated. Under our strong assumption that the volatility is uncorrelated with the aggregate consumption, this pricing equation is also true in a risky world. If we can derive the analytic form for the distribution of $\bar{V}$ with reasonable assumptions about the process driving V, then we can compute the bank equity value in equation (14) which is just the Black-Scholes price integrated over the distribution of the mean volatility.

Finally, based on the put-call parity, we can compute the value of deposit insurance[6], G(A_t, σ^2, T-t; B), as follows.

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6 In order to apply the put-call parity, we follow the tradition of Marcus and Shaked (1984), Ronn and Verma(1986) and Pennacchi (1987) and assume the put option can be exercised only at the maturity of the contract when insurer audits the bank.

$$G(A_t, V_t, T\text{-}t; B) = E(A_t, V_t, T\text{-}t; B) - A_t + Be^{-r(T-t)} \quad (15)$$

where $E(A_t, V_t, T\text{-}t; B) = \int C(\bar{V})\, g(\bar{V} \mid \sigma^2) d\bar{V}$ and

$$C(\bar{V}) = A_t N(d_1) - B\, e^{-r(T-t)} N(d_2)$$
$$d_1 = \log(A_t/B) + (r + (\bar{V}/2))\,(T\text{-}t)$$
$$d_2 = \log(A_t/B) + (r - (\bar{V}/2))\,(T\text{-}t)\ .$$

Equation (15) gives us the value of deposit insurance premium derived by the option pricing model under the assumption that the volatility of the bank assets is stochastic.

(B). When the Volatility is Correlated with the Market Value of Bank Assets ($\rho \neq 0$)

After deriving the valuation formula for the bank equity and deposit insurance in the case of zero correlation between the volatility and the market value of bank assets, we relax the assumption by allowing the volatility to be correlated with the bank assets value. From the previous derivation, the value of bank equity should satisfy the following differential equation given that the volatility is uncorrelated with aggregate consumption.

$$(\partial E/\partial t) + (1/2)[\sigma^2 A^2(\partial^2 E/\partial A^2) + 2\rho\,\sigma^3 \xi\, A(\partial^2 E/\partial VA\,\partial V)$$
$$+ \xi^2 V^2(\partial^2 E/\partial V^2) - rE$$
$$= -rA\,(\partial E/\partial A) - \mu_V \sigma^2(\partial E/\partial V) \quad (5)$$

Since the volatility V and the bank assets value are now instantaneously correlated. The property of the $\log\{A_T/A_0\}$ can not hold anymore. To see this, let A_j be the bank assets value at the end of ith period. Assume the variance changes at only n equally spaced times in the interval of two consecutive auditing time. Let V_{j-1} be the volatility during the i period. Then, $\log\{A_j/A_{j-1}\}$ and $\log\{V_j/V_{j-1}\}$ are normal distributions that in the limit have correlation ρ. So,the distribution of $\log\{A_T/A_0\}$ conditional on the path followed by V has a normal distribution with mean and variance depending on the attributes of the path followed by V other than $\bar{V}$. Thus, we can not determine the term inside the bracket of equation (11)

$$E(A_t, V_t, t) = \int e^{-r(T-t)} \int E(A_T)\, h(A_T, V_T \mid \bar{V})\, dA_T]g(\bar{V} \mid \sigma^2) d\bar{V}$$

and there will not be analytic form for the solution of the bank equity value. By the same argument as before, we can apply the put-call parity to compute the value of deposit insurance based on the bank equity value, i.e., the value of deposit insurance, $G(A_t, \sigma^2, T-t; B)$, will be

$$G(A_t, V_t, T\text{-}t; B) = E(A_t, V_t, T\text{-}t; B) - A_t + Be^{-r(T-t)} \quad (16)$$

where $E(A_t, V_t, t) = \int e^{-r(T-t)} \int E(A_T)\, h(A_T, V_T \mid \bar{V}) dA_T]$
$g(\bar{V} \mid \sigma^2) d$

and $E(A_T) = \mathrm{Max}(V_T - B, 0)$

Now, we look at one special case when the volatility is perfectly correlated with the bank equity value.

Case 1: $\rho = 1$ (The volatility is perfectly positively correlated with the market value of bank's assets)

In this case, the bank equity value should satisfy the following differential equation

$$(\partial E/\partial t) + (1/2)[\sigma^2 A^2(\partial^2 E/\partial A^2) + 2\sigma^3 \xi A(\partial^2 E/\partial A\, \partial V) + \xi^2 V^2(\partial^2 E/\partial V^2)] - rE = -rA(\partial E/\partial A) - \mu_V \sigma^2(\partial E/\partial V) \quad (17)$$

Case 2: $\rho = -1$ (The volatility is perfectly negatively correlated with the market value of bank's assets)

In this case, the bank equity value should satisfy the following differential equation

$$(\partial E/\partial t) + (1/2)[\sigma^2 A^2(\partial^2 E/\partial A^2) - 2\sigma^3 \xi A(\partial^2 E/\partial A\, \partial V)$$

$$+ \xi^2 V^2(\partial^2 E/\partial V^2)]\text{-rE}$$
$$= \text{-rA}\ (\partial E/\partial A) - \mu_V \sigma^2(\partial E/\partial V) \qquad (18)$$

In Case 1, the bank equity value can be solved from equation (19) in addition to satisfying equations (17). In Case 2, the bank equity value can be solved from equation (19) in addition to satisfying equation (18).

$$E(A_t, V_t, t) = \int e^{-r(T-t)} \int E(A_T)\ h(A_T, V_T | \forall) dA_T] g(\forall \mid \sigma^2) d\forall \qquad (19)$$

where $E(A_T) = \text{Max}(V_T\text{-B}, 0)$.

Again, the value of the deposit insurance is

$$G(A_t, V_t, T\text{-}t; B) = E(A_t, V_t, T\text{-}t; B) - A_t + Be^{-r(T-t)} \qquad (20)$$

III. COMPARISON OF MERTON'S VS. STOCHASTIC MODEL FOR PRICING THE COST OF DEPOSIT INSURANCE

In this section, we demonstrate that Merton's (1977) cost of deposit insurance model is simply a special case of our generalized mode when the volatility is assumed to be nonstochastic. To show this relation exists between the two models, we can first rewrite Merton's pricing formula in our notations as follows (see equations (19.5) and (19.6) in Merton (1977)). As in our text, Merton (1977) assumed the current value of the insured deposits when both principal and interest are guaranteed is

$D = Be^{-r(T-t)}$.

Merton (1977) showed that the pricing formular for the cost of the guarantee per dollar of insured deposits, $\Gamma\ (D/A_t, T\text{-}t)$, can be presented as[7]

$$\Gamma\ (D/A_t,\ T\text{-}t) \equiv \Gamma\ (D/A_t,\ T\text{-}t) = N(-h_2) - (A_t/D)[N(-h_1)]$$
$$\text{where}\quad h_1 \equiv [\ln(D/A_t) + 0.5\,\sigma\,(T\text{-}t)^{1/2}]/[\,\sigma\,(T\text{-}t)^{1/2}],$$
$$h_2 \equiv h_1 - \sigma\,(T\text{-}t)^{1/2},$$

and D/A_t is the current deposit-to-asset value ratio.

In the previous sections, we derived the generalized form for the cost of deposit insurance with stochastic volatility and the total cost of the guarantee can be stated as

$$G(A_t,\ V_t,\ T\text{-}t;\ B) = E(A_t,\ V_t,\ T\text{-}t;\ B) - A_t + Be^{-r(T-t)}$$

If we assume zero correlation between stock's return and it's volatility(ρ =0), the cost of deposit insurance function can be simplified as

$$G(A_t,\ V_t,\ T\text{-}t;\ B) = E(A_t,\ V_t,\ T\text{-}t;\ B) - A_t + Be^{-r(T-t)}$$
$$\text{where}\quad E(A_t,\ V_t,\ t;\ B) = \int C(\bar{V})\ \Gamma\ (\bar{V} \mid \sigma^2)d\bar{V}$$
$$\text{and}\quad C(\bar{V}) = A_t\ N(d_1) - Be^{-r(T-t)}\ N(d_2)$$
$$d_1 = [\log(A_t/B) + (r + (\bar{V}/2))\ (T\text{-}t)]/[\bar{V}\,(T\text{-}t)]^{1/2}$$
$$d_2 = [\log(A_t/B) + (r - (\bar{V}/2))\ (T\text{-}t)]/\ [\bar{V}\,(T\text{-}t)]^{1/2}$$
$$\bar{V} = [1/(T\text{-}t)]\ \int_t^T \sigma_s^2\ ds.$$

If we further assumed that volatility is nonstochatic, the generalized form will be degenerated into Merton's special case. If V is constant, then

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7 When we apply put option concept in the deposit insurance, we must interpret the contract with some caution. Unlike put option contraacts on securities, there is no incentive for the owner (the bank) of aa deposit insurance put option to exercise the option voluntarily until required to do so by the writer of the put option (the insurance agency).

$$E(A_t, V_t, T\text{-}t; B) = E(A_t, T\text{-}t; B) = A_tN(d_1) - Be^{-r(T-t)}N(d_2)$$

$$\text{where } d_1 = [\log(A_t/B) + (r + (\sigma^2/2))(T\text{-}t)] / \sigma (T\text{-}t)^{1/2}$$

$$d_2 = [\log(A_t/B) + (r - (\sigma^2/2))(T\text{-}t)] / \sigma (T\text{-}t)^{1/2}.$$

Hence, the total guarantee cost of deposit insurance can be rewritten as

$$\begin{aligned} G(A_t, T\text{-}t; B) &= E(A_t, T\text{-}t; B) - A_t + Be^{-r(T-t)} \\ &= A_t N(d_1) - Be^{-r(T-t)} N(d_2) - A_t + Be^{-r(T-t)} \\ &= D [1-N(d_2)] - A_t [1-N(d_1)] \\ &= D N(-d_2) - A_t N(-d_1)] \end{aligned}$$

$$\begin{aligned} \text{and } d_1 &= [\ln(A_t/D) - r(T\text{-}t) + (r + 0.5\sigma^2)(T\text{-}t)] / \sigma (T\text{-}t)^{1/2} \\ &= [\ln(A_t/D) + 0.5\sigma^2 (T\text{-}t)] / \sigma (T\text{-}t)^{1/2} \\ &= h_1 \end{aligned}$$

$$\begin{aligned} d_2 &= [\ln(A_t/D) - r(T\text{-}t) + (r - 0.5\sigma^2)(T\text{-}t)] / \sigma (T\text{-}t)^{1/2} \\ &= [\ln(A_t/D) - 0.5\sigma^2(T\text{-}t)] / \sigma (T\text{-}t)^{1/2} \\ &= h_2 \end{aligned}$$

The cost of the guarantee per dollar of insured deposits can be derived strightforward simply by dividing the total cost by the amount of deposits, that is

$$\begin{aligned} \Gamma (D/A_t, T\text{-}t) &= G(A_t, T\text{-}t; B)/D \\ &= N(-d_2) - (A_t/D)[N(-d_1)] \\ &= N(-h_2)-(A_t/D)[N(-h_1)] \end{aligned}$$

(Merton's special model)

where $\Gamma (D/A_t, T\text{-}t)$ is the cost of the guarantee per dollar of insured deposits, and (A_t/D) is the current asset-to-deposit ratio.

The above analysis of both Merton's and our model should be interpreted carefully for the following reasons[8]. (1) The price do not consider the bank's dynamic "gambling" incentive at low or negative values of net worth, or that the economic behavior of the owner of the put is endogenous and affected by the strike price. (2) The put price does not consider that the insurance agency may not accept delivery of the bank at maturity for a European option or at the time the strike price is reached for an American option and instead grants forbearance which can increase its losses further. (3) If the strike price is set at a sufficiently negative net worth value, the cost of the put to the insurer may be greater than the value of the put to the bank shareholders.(Kaufman[1992])

IV. Concluding Remarks

In this paper, we followed Merton's (1977) approach and derived the valuation model for deposit insurance in which volatility of the return of the underlying asset is stochastic. This is done by applying the recent development of option pricing model with stochastic volatility to the valuation of deposit insurance premiums. Based on Hull and White's (1987) findings, the nonstochastic Black-Scholes price always overprices at- the-money options, but underprices options that are sufficiently deeply in- or out-of-the- money. Hence, we expect risk-based deposit insurance premiums estimates obtained from our option model with stochastic volatility will be higher than

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8 The separation between the time the strike price is reached and the option is ready to be exercised and the time the insurer is willing to accept delivery represents an important component "forbearance risk". Hence, the option may be modelled as a compound put-call option instead of standard option pricing model (see Pennacchi[1986] and Kaufman[1992]).

those from option models with nonstochastic volatility assumption. The empirical findings of Marcus and Shaked (1984), Ronn and Verma (1986) and Pennachi (1987) that the U.S. deposit insurer overcharged the insured depository institutions are therefore unwarranted. Their estimates are obtained under a very restricted model which is only a special case of our general model. Once we allow the volatility of the underlying asset to be stochastic, new estimates of deposit insurance premiums should be higher than their original esitimates in which volatility is assumed as nonstochastic. Consequently, U.S. deposit insuer might not overcharge its insured clients at all .

Quiet possibly, our generalized model can resolve the empirical controversy between the overcharged insurance premiums and the insolvent condition of insurance funds by providing a model that can estimate the insurance premiums more correctly. Our model may also produce estimates that can reconcile the theoretical proposition argued by Buser, Chen and Kane (1981) that deposit insurer undercharged the deposit insurance premiums.

While various research of deposit insurance are critical, the pricing issue of risk- based insurance premiums is the cornerstone that a sound deposit insurance system needs. Only with a correctly priced risk-based premium, we can restrict the incentives of risk-taking and avoid the subsidization of high-risk bank by penalizing conservatively- run banks. Further empirical study to estimate deposit insurance premiums with our model would therefore be particularly important and interesting to be performed. Perhaps the most challenging work of this empirical study would be the estimation of stochatastic volatility.

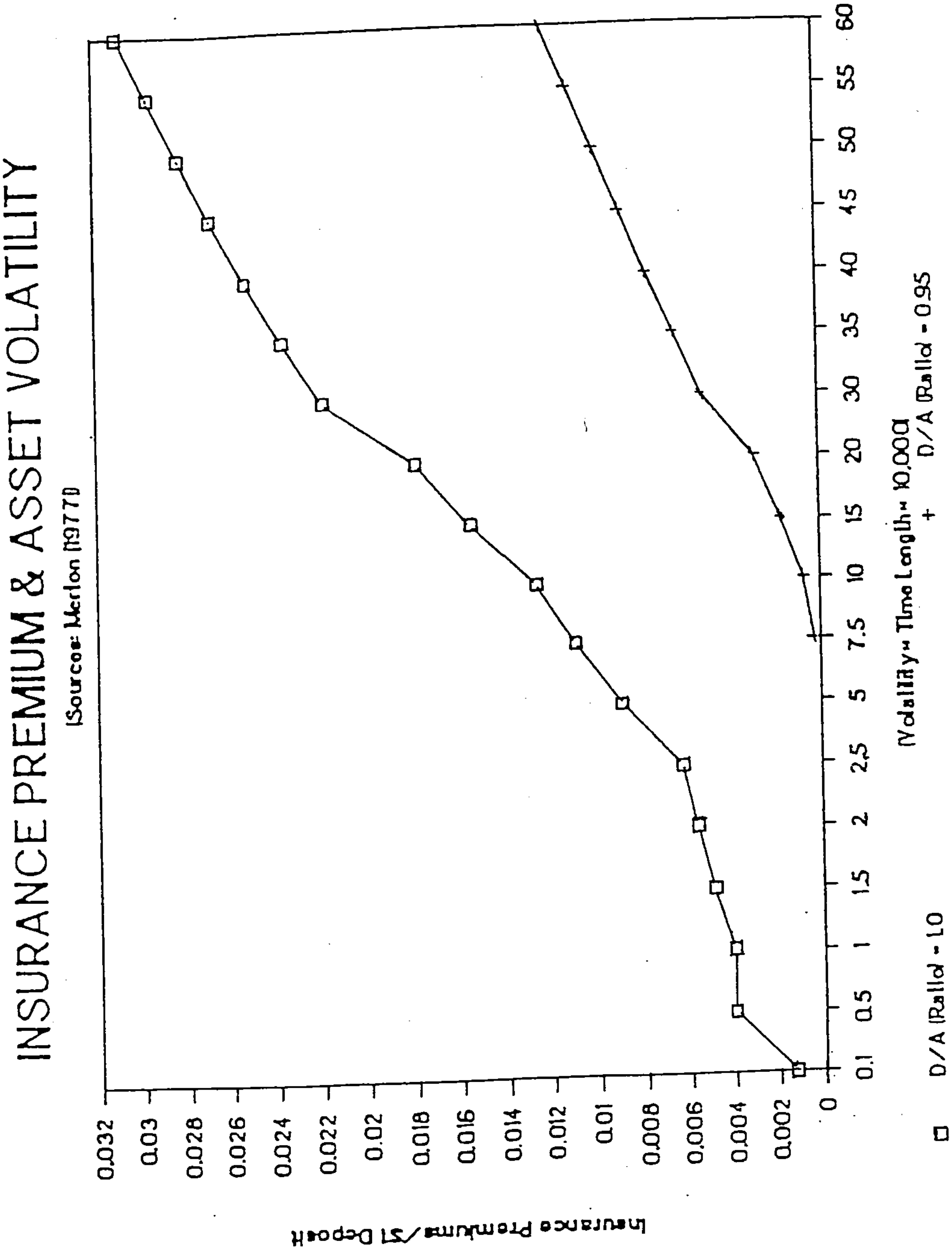
INSURANCE PREMIUM & ASSET VOLATILITY
(Source: Merton (1977))
Insurance Premiums/$1 Deposit
0.032
0.03
0.028
0.026
0.024
0.022
0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0.1
0.5
1
1.5
2
2.5
5
7.5
10
15
20
30
35
40
45
50
55
60
(Volatility* Time Length* 10,000)
D/A (Ratio) = 1.0
D/A (Ratio) = 0.95

Figure 1

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存款保險成本的評價：隨機變異值選擇權訂價模型的應用

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摘　要

本文首度應用隨機變異值選擇權訂價模型到存款保險費率的評價上。我們比較 Merton(1977) 模型及隨機變異值模型後發現，Merton 模型只是我們的一般化模型之特例，由於 Black-Scholes 的標準選擇權模型經常低估深價內（deep in-the-money) 以及遠價外（deep out-of-the-money）的情形，我們預期未來應用此隨機變異值模型的實證研究將可獲得較高的存款保險費率估計值，因而可以化解一方面存保費率過度徵收的實證發現而另一方面 FDIC 的存款保險基金卻又不足的現象。

關鍵字：存款保險費率、Black-Scholes 選擇權定價模型、隨機變異數、隨機微分方程。