OPTIMAL QUALITY STRATEGY IN MARKOVIAN RATING SYSTEM

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Abstract

Assume a supplier/customer management chain has a quality rating system. In this system, a supplier who did not make any nonconforming lot to his customer during one period are entitled to a discount membership. If he declared one or more nonconforming lots to the customer, then he is penalized by an increase of membership. This paper describes how to compute the optimal strategy for quality rating system. The strategy determines the supplier whether or not to declare a nonconforming lot to his customer. The model contains stochastic process, Markovian property, and iterative algorithm.

Key Words : Quality Rating System, Gaussian Elimination, Markovian Property, Optimal Strategy.

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I. Introduction

Assume a supplier/customer management chain has a quality rating system. In this system, a supplier who did not make any nonconforming lot to his customer during one period are entitled to a discount membership. If he declared one or more nonconforming lots to the customer, then he is penalized by an increase of membership. Each nonconforming lot has a loss amount. The loss amount is something like the Taguchi's loss function, but in this paper we define loss amount as a random variable. Suppose the periodic membership paid by the supplier depends on the number of nonconforming lots declared to the customer, but not on the loss amounts of the nonconforming lot. This system naturally induces a supplier, involved in a nonconforming lot which has small loss amount, to pay by himself the loss amount and not to declare the nonconforming lot to his customer, in order to avoid a future increase of his membership. The question is how much is the critical point for a supplier to declare or not to declare the nonconforming lot. We shall attempt to determine the optimal strategy of the supplier in this rating system. Figure 1 illustrate the rating system.

The remainder of this paper is organized as follows. In section II, we formulate the problem as a stochastic process model, starting with a initial strategy, we apply a iterative algorithm to converge to an optimal strategy. Section III illustrate an example with computation results. In section IV, we extend the problem to the absorbing Markovian model. Conclusions are presented in section V.

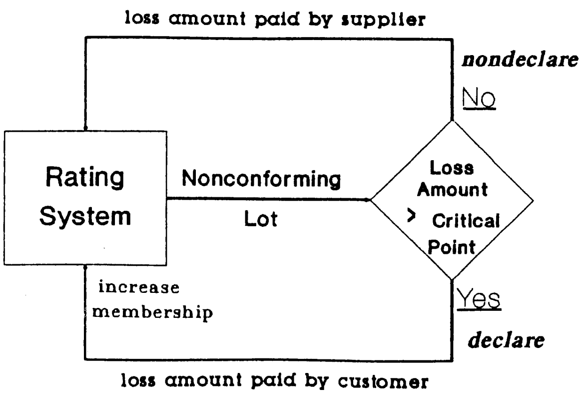

Figure 1 Quality Rating System

II. Problem Formulation

We assume that a supplier/customer chain uses a rating system. The suppliers can be partitioned into a finite number of classes C_i $(i = 1, \ldots, s)$, in such a way that the periodic membership paid by the supplier only depends on the class. The class at a given period is uniquely determined by the class of the preceding period and the number of nonconforming lots declared to the customer during the last period. This is a Markovian property.

Such a system can be defined by :

(i) The membership scale vector $b = (b_1, \ldots, b_s)$ where b_i denotes the membership level when the present period in class C_i ;

(ii) The transition rules, which are the laws governing the passages from one class to another class when the number of nonconforming lots is known. The rules can be defined as a set of transformations T_k, such that $T_k(i) = j$, the policy is transferred from class C_i to C_j if k nonconforming lots are made during the period.

Let $\{p_\lambda(k) \mid k = 0, \ldots\}$ be the distribution of the number of nonconforming lots for one period of this supplier, we assume this is Poisson distribution where λ is his expected number per period. We can assume that the supplier will pay by himself the small loss of nonconforming lot, below a certain limit $\vec{x}$, and declare any nonconforming lot loss amount above $\vec{x}$. This retention limit must depend on the class C_i where the supplier belong to, so we shall denote it x_i. A strategy of the supplier is then a vector $\vec{x} = (x_1, \ldots, x_s)$.

If X is the random variable representing the loss amount of a nonconforming lot, $f(x)$ is the probability density function of continuous X, or $P(X = k)$ is the probability mass function of discrete X.

We assume that the distribution of the number of nonconforming lots for one period of this supplier and the distribution of the loss amount of a nonconforming lot are independent.

The probability that a given nonconforming lot will **not** be declared to the customer equals

$$p_i = P\{X \le x_i\} = \int_0^{x_i} f(x)dx, \qquad i = 1, ..., s.$$

$$p_i = P\{X \le x_i\} = \sum_{k=0}^{x_i} P(X = k), \qquad i = 1, ..., s.$$

where x_i is any retention limit.

The probability $p_\lambda(i,k)$ that k nonconforming lots will be made during a period for a given class C_i equals

$$p_\lambda(i,k) = \sum_{h=k}^{\infty} p_\lambda(h)\binom{h}{k}(1-p_i)^k(p_i)^{h-k}, \qquad i = 1, ..., s\ ;\ k = 0, ..., \infty.$$

where $\binom{h}{k}$ is the binomial coefficient.

The mathematical expectation of the number of declared nonconforming lots equals

$$m_i = \sum_{k=0}^{\infty} kp_\lambda(i,k), \qquad i = 1, ..., s.$$

The cost expectation of a non-declared nonconforming lot given x_i is

$$E_i = E_{x_i} = E[X \mid X \le x_i] = \left(\frac{1}{p_i}\right)\int_0^{x_i} xf(x)\,dx, \qquad i = 1, ..., s.$$

$$E_i = E_{x_i} = E[X \mid X \le x_i] = \left(\frac{1}{p_i}\right)\sum_{k=0}^{x_i} kP(X = k), \qquad i = 1, ..., s.$$

The average non-declared nonconforming lot payment by the supplier for one period is

$$E(x_i) = b_i + \sqrt{q}(\lambda - m_i)E_i, \qquad i = 1, ..., s.$$

where q is a discount factor and compromise all the nonconforming lots in the middle of the period, so we take the square of q.

Let V_i be the discounted expectation of all the future costs of a supplier who is in class C_i at the beginning of a period. V_i must satisfy the system

$$V_i = E(x_i) + q \sum_{k=0}^{\infty} p_\lambda(i,k) V_{T_k(i)}, \qquad i = 1, ..., s. \tag{1}$$

This is a system of s equations with s unknowns. We can solve these simultaneous equations by Gaussian elimination.

The supplier responsible of a nonconforming lot of cost x must choose between two actions : to declare the nonconforming lot or not to declare the nonconforming lot.

If he does not declare the nonconforming lot , the present value of the total expectation of all his cost is

$$E(x_i) + x + q \sum_{k=0}^{\infty} p_\lambda(i,k) V_{T_{n+k}(i)} \tag{2}$$

where n is the number of nonconforming lots already declared during the present period.

If the nonconforming lot is declared, the total future expectation cost becomes

$$E(x_i) + q \sum_{k=0}^{\infty} p_\lambda(i,k) V_{T_{n+k+1}(i)} \tag{3}$$

The retention limit x_i is of course the x in (2) for which both expectation (2) and (3) are equivalent.

$$x_i = q \sum_{k=0}^{\infty} p_\lambda(i,k)[V_{T_{n+k+1}(i)} - V_{T_{n+k}(i)}], \qquad i = 1, \ldots, s. \tag{4}$$

This is a system of s equations with s unknowns x_i. It can be shown that this system has one and only one solution, for a given vector $V = (V_1, \ldots, V_s)$. So the optimal strategy $\vec{x}^* = (x_1^*, \ldots, x_s^*)$ can be obtained by successive approximations, applying (1) and (4) by iteration. We can start the algorithm by choosing any strategy, for instance $\vec{x}^0 = (0, \ldots, 0)$ which declare all the nonconforming lot.

Given the first strategy, we can apply the system (1) to determine a first value vector V^0. An improved strategy $\vec{x}^1$ can be found by applying system (4). Successive applications of (1) and (4), $\{\vec{x}^0, V^0, \vec{x}^1, V^1, \vec{x}^2, V^2, \ldots\}$ may converge to the optimal strategy $\vec{x}^*$ and V^*.

The algorithm as following :

Step 0 : Define number of class s, determine class C_i, b_i, λ, $f(x)$ or $P(X = k)$

let $k = 0$, $\vec{x}^k = (x_1^k, \ldots, x_s^k) = (0, \ldots, 0)$

Step 1 : Compute p_i , $p_\lambda(i,k)$, m_i , E_i , $E(x_i^k)$

Step 2 : Solve these simultaneous equations :

$$V_i = E(x_i^k) + q\sum_{k=0}^{\infty} p_\lambda(i,k)V_{T_k(i)}, \qquad i = 1, \ldots, s.$$

Let the solution be $V^k = (V_1^k, \ldots, V_s^k)$.

Step 3 : Apply $V^k = (V_1^k, \ldots, V_s^k)$ to equations :

$$x_i = q\sum_{k=0}^{\infty} p_\lambda(i,k)[V^k_{T_{k+1}(i)} - V^k_{T_k(i)}], \qquad i = 1, \ldots, s.$$

Let the solution be $\vec{x}^{k+1} = (x_1^{k+1}, \ldots, x_s^{k+1})$

(Note : In equation (4), there is a $n = 0$ in $V_{T_{n+k}(i)}$. But from the computation results, We found that n is not sensitive to the optimal strategy. So we take $n = 0$ in this step.)

Step 4 : If $||V^k - V^{k-1}|| < \delta$, for some δ, then goto Step 6.

Else let $k = k + 1$, goto Step 5.

Step 5 : If $k > 1000$, then the problem do not converge to optimal strategy, stop.

Else , goto Step 1.

Step 6 : $\vec{x}^* = \vec{x}^k$, $V^* = V^k$

Establish a transition probability matrix $\mathbf{P} = (p_{ij})$

$$p_{ij} = \sum_{k:j=T_k(i)} P_\lambda(i,k) \qquad i = 1,..,s \; ; \; j = 1,...,s$$

then compute the steady-state probability $\mathbf{a}^* = (\mathbf{a}_1^*, ..., \mathbf{a}_s^*)$

for each class.

Step 7 : Solution validation by simulation :

Use $\vec{x}^*$ to simulate 10000 periods.

compute p^* = the probability of non-declared nonconforming lot under the optimal strategy.

compute m^* = the declared nonconforming lot frequency under the optimal strategy.

compute $E(x^*)$ = the expectation of one period payments.

compute $\mathbf{a}^*$ = the stationary probability distribution under the the optimal strategy.

compare above results with step 6.

III. Application To An Example

Suppose a rating system consists of 18 classes with periodic memberships in Table 1. For example, a new supplier enters the system in class 6. For each no-nonconforming period, the supplier reduce one class. The penalty are 2 classes for the first nonconforming lot, 3 classes for each successive nonconforming lot. That is $T_0(i) = i - 1$, $T_1(i) = i + 2$, $T_2(i) = i + 2 + 3$, $T_3(i) = i + 2 + 3 + 3$,etc.

There is one addition to those rules : a supplier who has four consecutive no-nonconforming periods, is automatically brought back to class 10 if he is above class 10. This restriction leads the system become non-Markovian.

Table 1 Classes and membership

Class	Membership (unit : $00)
1	55
2	60
3	65
4	70
5	75
6	80
7	85
8	90
9	95
10	100
11	105
12	110
13	115
14	120
15	130
16	140
17	160
18	200

To restore the Markovian property we have to subdivide some of the classes, adding a digit indicating the number of no-nonconforming periods. The new process is Markovian. It has 30 classes (see Table 2).

For example : class 12.3 means that now is in class 12 and has three consecutive no-nonconforming periods.

We shall make the following assumptions for specific supplier :

1. The probability distribution of the number of responsible nonconforming lots of the supplier is Poisson with the parameter $\lambda = 1$.

Table 2 Classes for Markovian property

Class	T_0	T_1	T_2	T_3	T_4	T_5	$T_k : k > 5$
1	1	3	6	9	12	15	18
2	1	4	7	10	13	16	18
3	2	5	8	11	14	17	18
4	3	6	9	12	15	18	18
5	4	7	10	13	16	18	18
6	5	8	11	14	17	18	18
7	6	9	12	15	18	18	18
8	7	10	13	16	18	18	18
9	8	11	14	17	18	18	18
10	9	12	15	18	18	18	18
11	10	13	16	18	18	18	18
12.3	10	14	17	18	18	18	18
12	11	14	17	18	18	18	18
13.3	10	15	18	18	18	18	18
13.2	12.3	15	18	18	18	18	18
13	12	15	18	18	18	18	18
14.3	10	16	18	18	18	18	18
14.2	13.3	16	18	18	18	18	18
14.1	13.2	16	18	18	18	18	18
14	13	16	18	18	18	18	18
15.3	10	17	18	18	18	18	18
15.2	14.3	17	18	18	18	18	18
15.1	14.2	17	18	18	18	18	18
15	14.1	17	18	18	18	18	18
16.2	15.3	18	18	18	18	18	18
16.1	15.2	18	18	18	18	18	18
16	15.1	18	18	18	18	18	18
17.1	16.2	18	18	18	18	18	18
17	16.1	18	18	18	18	18	18
18	17.1	18	18	18	18	18	18

2. The distribution of the loss amount of one nonconforming lot is a random variable X with the following distribution (unit : $00) :

$$P(X=5)=\frac{1}{45},\ P(X=8)=\frac{2}{45},\ P(X=10)=\frac{3}{45},\ P(X=30)=\frac{4}{45}$$

$$P(X=50)=\frac{5}{45},\ P(X=70)=\frac{7}{45},\ P(X=80)=\frac{6}{45},\ P(X=100)=\frac{5}{45}$$

$$P(X=150)=\frac{4}{45},P(X=200)=\frac{3}{45},P(X=300)=\frac{2}{45},\ P(X=500)=\frac{1}{45}$$

3. The discount rate is $1/(1+6\%)$.

The results are summarized in the Table 3.

Column (1) : C_i is the class.

Column (2) : $\vec{x}^*$ is the optimal strategy of the supplier for each class.

Column (3) : V^0 is the expectation of all future costs when one declares every nonconforming lot.

Column (4) : V^* is the expectation of all future costs under the optimal strategy.

Column (5) : p^* is the probability of non-declared nonconforming lot under the optimal strategy.

Column (6) : m^* is the declared nonconforming lot frequency under the optimal strategy.

Column (7) : $E(x^*)$ is the expectation of one period payments.

Column (8) : a^* is the stationary probability distribution under the the optimal strategy.

We note that for classes C_i = 14.3, 14.2, 14.1, 14 ; $x^*_{14.3}$ = 283.5, $x^*_{14.2}$ =215.4, $x^*_{14.1}$ =157.3, x^*_{14} =120.8. This means that in one particular class, as the number of nondeclared-nonconforming periods increase, then the retention limit is increase.

Table 3 Optimal strategy , nonconforming lot frequency and stationary probability †‡

(1) C_i	(2) $\vec{x}^*$	(3) V^0	(4) V^*	(5) p^*	(6) m^*	(7) $E(x^*)$	(8) a^*
1	79.4	2527	1834	.35	.65	64.5	.1806
2	88.7	2558	1850	.65	.35	91.4	.1225
3	103.6	2600	1875	.77	.23	108	.1546
4	119.8	2644	1909	.77	.23	113	.1177
5	133.0	2689	1949	.77	.23	118	.1104
6	143.5	2733	1994	.77	.23	123	.1034
7	150.4	2776	2043	.86	.14	141	.0710
8	157.4	2818	2096	.86	.14	146	.0522
9	162.6	2858	2151	.86	.14	151	.0343
10	164.4	2898	2209	.86	.14	156	.0216
11	163.3	2936	2268	.86	.14	161	.0125
12.3	214.9	2957	2279	.93	.07	180	.0006
12	159.2	2971	2328	.86	.14	166	.0067
13.3	249.0	2982	2286	.93	.07	185	.0003
13.2	183.6	3003	2347	.86	.14	171	.0007
13	140.8	3008	2388	.77	.23	158	.0032
14.3	283.5	3001	2294	.93	.07	190	.0002
14.2	215.4	3030	2362	.93	.07	190	.0004
14.1	157.3	3037	2413	.86	.14	176	.0008
14	120.8	3039	2444	.77	.23	163	.0013
15.3	331.9	3022	2305	.98	.02	213	.0004
15.2	244.8	3058	2381	.93	.07	200	.0003
15.1	176.4	3068	2440	.86	.14	186	.0004
15	127.2	3071	2481	.77	.23	173	.0001
16.2	305.2	3093	2405	.98	.02	223	.0004
16.1	224.1	3105	2472	.93	.07	210	.0003
16	146.4	3109	2523	.77	.23	183	.0005
17.1	189.6	3138	2513	.86	.14	216	.0004
17	122.3	3142	2567	.77	.23	203	.0003
18	60.3	3193	2636	.34	.66	210	.0009

† The units for $\vec{x}^*$, V^0 , V^* and $E(x^*)$ are \$00

‡ p^* , m^* and a^* are probabilities.

Figure 2 shows the optimal strategy of the supplier for each class $\vec{x}^*$ and the expectation of all future costs under the optimal strategy V^*.

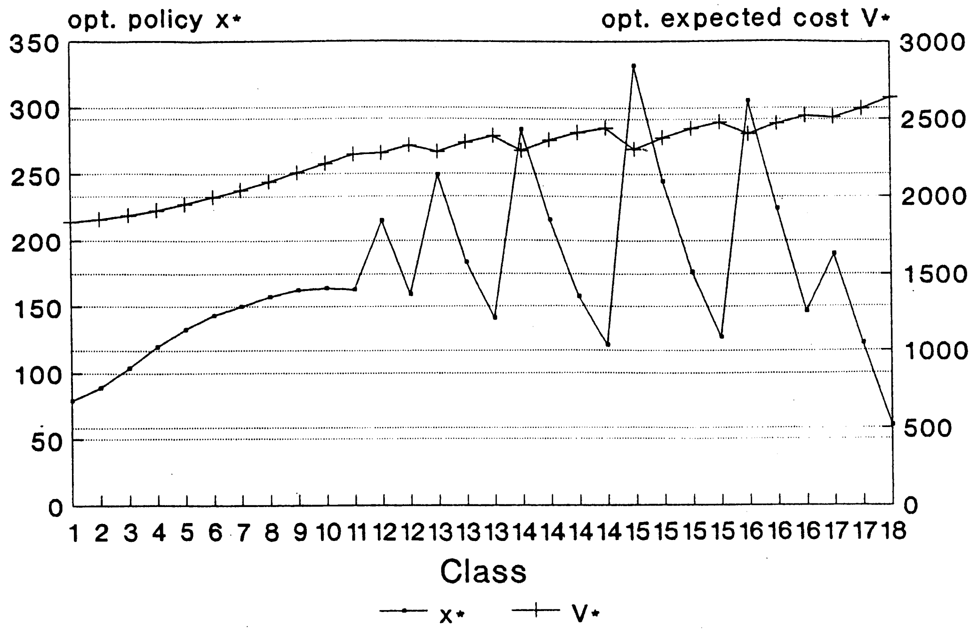

Figure 2 Optimal policy and cost

Table 4 and figure 3 show the optimal strategy for different average number of nonconforming lots. λ = 0.5, 1.0, 1.5, 2.0, 3.0, 4.0, 5.0, 10.0

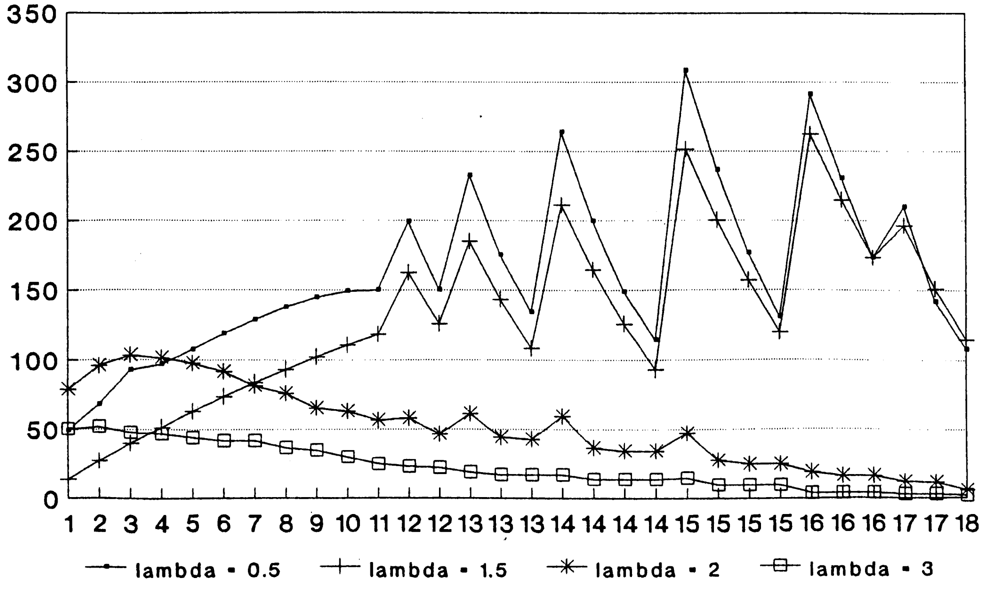

Figure 3 Optimal Strategy for lambdas

We find that as the average number of nonconforming lots per period λ increase, the optimal strategy $\vec{\mathbf{x}}^*$ decrease. For $\lambda = \mathbf{10}$, $\vec{\mathbf{x}}^* \approx \mathbf{0}$, This means that the supplier has to report every nonconforming lots to the customer.

Table 4 Optimal strategy $\vec{\mathbf{x}}^*$ for different nonconforming lots frequency (unit : $00)

Class	$\lambda = .5$	$\lambda = 1$	$\lambda = 1.5$	$\lambda = 2$	$\lambda = 3$	$\lambda = 4$	$\lambda = 5$	$\lambda = 10$
1	49.5	79.4	13.9	78.8	50	36	26	3
2	67.97	88.7	27.3	95.8	52	32	24	2
3	92.3	103.6	39.9	103	48	31	23	2
4	96.54	119.8	51.8	101	47	31	22	1
5	107.1	133.0	63.0	97.6	44	38	20	1
6	118.4	143.5	73.6	91.4	42	26	18	1
7	128.4	150.4	83.6	81.4	42	25	16	.5
8	137.2	157.4	93.0	75.7	37	22	15	.4
9	144.5	162.6	101.9	65.2	35	20	11	.3
10	148.7	164.4	110.2	63.2	30	17	7	.1
11	150	163.3	118.2	56.6	25	14	6	.1
12.3	199.1	214.9	162.4	58.0	23	11	4	.1
12	149.9	159.2	125.6	46.7	22	10	4	.1
13.3	232.4	249.0	184.5	61.4	19	6	2	.03
13.2	175.1	183.6	142.9	44.3	17	6	2	.03
13	133.6	140.8	108.2	42.8	17	6	2	.03
14.3	263.9	283.5	211.0	59.4	17	5	2	.02
14.2	199.3	215.4	164.7	37.1	14	4	2	.02
14.1	148.3	157.3	125.6	34.8	14	4	2	.02
14	114.0	120.8	92.8	34.5	14	4	2	.02
15.3	308.8	331.9	251.1	47.6	15	4	2	.01
15.2	236.3	244.8	200.1	27.6	10	3	1	.01
15.1	176.3	176.4	156.5	25.1	10	3	1	.01
15	130.1	127.2	119.6	24.8	10	3	1	.01
16.2	291.3	305.2	262.5	19.1	4	1	.4	.00
16.1	230	224.1	214.5	16.4	4	1	.4	.00
16	173.3	146.4	173.3	16.0	4	1	.4	.00
17.1	209.6	189.6	196.1	12	3	1	.3	.00
17	141.7	122.3	150.8	11.5	3	1	.3	.00
18	107.7	60.3	114.6	5.9	2	1	.2	.00

Now we suppose that $\lambda = 1$, the distribution of the loss amount of one nonconforming lot is the Normal distribution $X \sim N(300, 100)$. Table 5 shows the optimal strategy of the supplier for each class $\vec{x}^*$.

Table 5 Optimal strategy and stationary probability for normal distribution of loss amount $X \sim N(300, 100)$

(1) C_i	(2) $\vec{x}^*$	(3) V^*	(4) p^*	(5) m^*	(6) $E(x^*)$	(7) a^*
1	101	2497.6	.0001	.9998999	53.31	.0089
2	115	2528.4	.0001	.9998999	58.31	.0049
3	118	2569.7	.0001	.9998999	63.31	.0075
4	117	2613.9	.0001	.9998999	68.31	.0018
5	114	2658.7	.0001	.9998999	73.31	.0048
6	110	2702.8	.0001	.9998999	78.31	.0060
7	106	2745.7	.0001	.9998999	83.31	.0054
8	103	2787.9	.0001	.9998999	88.31	.0071
9	98	2828.2	.0001	.9998999	93.31	.0101
10	95	2867.7	.0001	.9998999	98.31	.0170
11	88	2905.9	.0001	.9998999	103.31	.0090
12.3	94	2927.5	.0001	.9998999	108.31	.0003
12	80	2940.7	.0001	.9998999	108.31	.0101
13.3	103	2952.4	.0001	.9998999	113.31	.0004
13.2	82	2973.0	.0001	.9998999	113.31	.0009
13	77	2977.7	.0001	.9998999	113.31	.0072
14.3	103	2970.7	.0001	.9998999	118.31	.0007
14.2	73	3000.1	.0001	.9998999	118.31	.0009
14.1	66	3007.3	.0001	.9998999	118.31	.0024
14	64	3008.9	.0001	.9998999	118.31	.0061
15.3	103	2992.1	.0001	.9998999	128.31	.0264
15.2	67	3027.8	.0001	.9998999	128.31	.0024
15.1	57	3038.1	.0001	.9998999	128.31	.0023
15	54	3040.6	.0001	.9998999	128.31	.0065
16.2	59	3063.1	.0001	.9998999	138.31	.0737
16.1	47	3075.5	.0001	.9998999	138.31	.0065
16	43	3079.1	.0001	.9998999	138.31	.0060
17.1	35	3107.7	.0001	.9998999	158.31	.1986
17	30	3112.0	.0001	.9998999	158.31	.0171
18	19	3163.2	.0001	.9998999	198.31	.5491

Table 6 presents the optimal strategy $\vec{x}^*$ for different nonconforming lots frequency and different normal distribution of loss amount.

Table 6 Optimal strategy $\vec{x}^*$ for different nonconforming lots frequency λ and different normal distribution of loss amount $N(\mu, \sigma^2)$

Class C_i	$\lambda = 1$ $\mu = 300$ $\sigma = 10$	$\lambda = 1$ $\mu = 300$ $\sigma = 30$	$\lambda = 1$ $\mu = 500$ $\sigma = 20$	$\lambda = 2$ $\mu = 300$ $\sigma = 10$	$\lambda = 3$ $\mu = 200$ $\sigma = 10$
1	101	101	101	61	42
2	115	115	115	63	40
3	118	118	118	61	38
4	117	117	117	59	38
5	114	114	114	57	35
6	110	110	110	54	33
7	106	106	106	53	33
8	103	103	103	49	29
9	98	98	98	45	26
10	95	95	95	44	24
11	88	88	88	38	20
12.3	94	94	94	37	18
12	80	80	80	34	17
13.3	103	103	103	34	14
13.2	82	82	82	30	13
13	77	77	77	30	13
14.3	103	103	103	31	12
14.2	73	73	73	24	11
14.1	66	66	66	24	10
14	64	64	64	24	10
15.3	103	103	103	28	10
15.2	67	67	67	20	8
15.1	57	57	57	19	8
15	54	54	54	19	8
16.2	59	59	59	13	4
16.1	47	47	47	12	4
16	43	43	43	11	4
17.1	35	35	35	9	3
17	30	30	30	8	3
18	19	19	19	5	2

IV. Absorbing Markovian Model Problem

We assume that the customer can withdraw the supplier if :

(1). The supplier has declared 6 or more nonconforming lots for one period.

(2). The supplier had 3 consecutive periods in the top class.

C_{s+1} is the withdraw class, that is an absorbing class in Markov model. A supplier enter the withdraw class, then he never come back to the rating system. The membership scale vector $b = (b_1, \ldots, b_s, b_{s+1})$, where $b_{s+1} = 0$.

The transition rules T_k, such that $T_k(i) = j$, are presented in Table 7. We denote class 19 as the withdraw class, and two addition class 18.1 and 18.2 as in class 18 for 1 and 2 consecutive periods.

Let S be the one period expected cost for the supplier in the withdraw class. Let W be the discounted expectation of all the future costs of a supplier who is in the withdraw class.

$$S = \lambda \int_0^{\infty} x f(x)\, dx.$$

$$W = \sum_{i=1}^{\infty} q^i S$$

If W diverges to ∞, we take a finite life N of rating system.

$$W = \sum_{i=1}^{N} q^i S$$

The following notations : p_i , $p_\lambda(i,k)$, E_i , $E(x_i)$, V_i are defined as Section II.

$$p_i = P\{X \le x_i\} = \int_0^{x_i} f(x)dx, \qquad i = 1, \ldots, s.$$

$$p_\lambda(i,k) = \sum_{h=k}^{\infty} p_\lambda(h)\binom{h}{k}(1-p_i)^k(p_i)^{h-k}, \qquad i = 1, \ldots, s \ ; \ k = 0, \ldots, \infty.$$

Table 7 Classes for Markovian property

Class	T_0	T_1	T_2	T_3	T_4	T_5	$T_k : k > 5$
1	1	3	6	9	12	15	19
2	1	4	7	10	13	16	19
3	2	5	8	11	14	17	19
4	3	6	9	12	15	18	19
5	4	7	10	13	16	18	19
6	5	8	11	14	17	18	19
7	6	9	12	15	18	18	19
8	7	10	13	16	18	18	19
9	8	11	14	17	18	18	19
10	9	12	15	18	18	18	19
11	10	13	16	18	18	18	19
12.3	10	14	17	18	18	18	19
12	11	14	17	18	18	18	19
13.3	10	15	18	18	18	18	19
13.2	12.3	15	18	18	18	18	19
13	12	15	18	18	18	18	19
14.3	10	16	18	18	18	18	19
14.2	13.3	16	18	18	18	18	19
14.1	13.2	16	18	18	18	18	19
14	13	16	18	18	18	18	19
15.3	10	17	18	18	18	18	19
15.2	14.3	17	18	18	18	18	19
15.1	14.2	17	18	18	18	18	19
15	14.1	17	18	18	18	18	19
16.2	15.3	18	18	18	18	18	19
16.1	15.2	18	18	18	18	18	19
16	15.1	18	18	18	18	18	19
17.1	16.2	18	18	18	18	18	19
17	16.1	18	18	18	18	18	19
18	17.1	18.1	18.1	18.1	18.1	18.1	19
18.1	17.1	18.2	18.2	18.2	18.2	18.2	19
18.2	17.1	19	19	19	19	19	19
19	19	19	19	19	19	19	19

$$m_i = \sum_{k=0}^{\infty} k p_\lambda(i,k), \qquad i = 1,...,s.$$

$$E_i = \left(\frac{1}{p_i}\right) \int_0^{x_i} x f(x)\, dx, \qquad i = 1,...,s.$$

$$E(x_i) = b_i + \sqrt{q}(\lambda - m_i)E_i, \qquad i = 1,...,s.$$

$$V_i = E(x_i) + q \sum_{k=0}^{5} p_\lambda(i,k) V_{T_k(i)} + q \sum_{k=6}^{\infty} p_\lambda(i,k) W, \qquad i = 1,...,s. \qquad (5)$$

If the supplier does not declare the nonconforming lot , the present value of the total expectation of all his cost is

$$E(x_i) + x + q \sum_{k=0}^{5} p_\lambda(i,k) V_{T_k(i)} + q \sum_{k=6}^{\infty} p_\lambda(i,k) W$$

If the nonconforming lot is declared, the total future expectation cost becomes

$$E(x_i) + q \sum_{k=0}^{4} p_\lambda(i,k) V_{T_{k+1}(i)} + q \sum_{k=5}^{\infty} p_\lambda(i,k) W$$

The retention limit x_i is of course the x for which both expectation are equivalent.

$$x_i = q \sum_{k=0}^{4} p_\lambda(i,k)[V_{T_{k+1}(i)} - V_{T_k(i)}] + q p_\lambda(i,5)[W - V_{T_5(i)}], \qquad i = 1,...,s. \qquad (6)$$

Let N_i be the expected number of periods that begin at class C_i, and end up in withdraw absorbing class.

$$N_i = \sum_{k=0}^{5} p_\lambda(i,k) N_{T_k(i)} \qquad (7)$$

We can use the same algorithm as Section II, apply equations (5) , (6) , (7) to find the optimal strategy $\vec{x}^*$, V^* and N^* for the absorbing Markovian model problem. The computation results are in Table 8 and Table 9.

Table 8 Optimal strategy , nonconforming lot frequency and transient time

$X \sim N(300, 100)$

(1) C_i	(2) $\vec{x}^*$	(3) V^*	(4) p^*	(5) m_i^*	(6) N^*
1	13.8	1129.3	.0001	.9999	22.6
2	27.2	1135.0	.0001	.9999	22.3
3	39.9	1140.9	.0001	.9999	21.9
4	52.0	1146.1	.0001	.9999	21.4
5	63.2	1149.1	.0001	.9999	20.9
6	73.6	1149.3	.0001	.9999	20.4
7	83.6	1148.9	.0001	.9999	19.9
8	92.9	1143.3	.0001	.9999	19.4
9	101.8	1137.6	.0001	.9999	18.9
10	110.1	1128.8	.0001	.9999	18.4
11	118.1	1113.3	.0001	.9999	17.9
12.3	162.4	1111.6	.0001	.9999	17.7
12	125.7	1106.2	.0001	.9999	17.5
13.3	184.3	1087.9	.0001	.9999	17.3
13.2	142.8	1081.9	.0001	.9999	17.0
13	108.1	1080.1	.0001	.9999	16.9
14.3	210.7	1076.0	.0001	.9999	17.0
14.2	164.4	1061.8	.0001	.9999	16.6
14.1	125.3	1059.7	.0001	.9999	16.5
14	92.57	1059.1	.0001	.9999	16.4
15.3	251.0	1086.1	.0001	.9999	16.9
15.2	200.1	1068.9	.0001	.9999	16.4
15.1	156.4	1063.9	.0001	.9999	16.2
15	119.6	1063.2	.0001	.9999	16.2
16.2	262.3	774.4	.0008	.9999	15.7
16.1	214.2	1016.2	.0001	.9999	15.5
16	173.2	1014.5	.0001	.9999	15.4
17.1	195.8	934.0	.0001	.9999	15.3
17	150.5	1017.9	.0001	.9999	15.2
18	0	844.6	.0001	.9999	13.4
18.1	0	734.2	.0001	.9999	10.8
18.2	0	654.9	.0001	.9999	6.6

Table 9 Optimal strategy $\vec{x}^*$ for different nonconforming lots frequency λ and different normal distribution of loss amount $N(\mu, \sigma^2)$

Class C_i	$\lambda = 1$ $\mu = 300$ $\sigma = 10$	$\lambda = 1$ $\mu = 300$ $\sigma = 30$	$\lambda = 1$ $\mu = 500$ $\sigma = 20$	$\lambda = 3$ $\mu = 300$ $\sigma = 10$	$\lambda = 1$ $\mu = 2000$ $\sigma = 100$
1	13.8	110.6	13.7	0	71
2	27.2	126.3	27.4	0	68
3	39.9	129.9	39.6	0	62
4	52.0	128.5	51.9	0	56
5	63.2	127.3	62.7	0	48
6	73.6	122.5	73.6	0	42
7	83.6	120.6	83.5	0	34
8	92.9	117.9	92.9	0	23
9	101.8	108.3	101.9	0	22
10	110.1	113.8	110.4	0	5
11	118.1	105.5	118.4	0	0
12.3	162.4	106.3	162.7	0	0
12	125.7	90.8	125.9	0	5
13.3	184.3	131.1	184.4	0	0
13.2	142.8	110.5	142.9	0	0
13	108.1	103.9	108.0	0	0
14.3	210.7	131.2	210.7	0	0
14.2	164.4	154.8	164.1	0	0
14.1	125.3	88.9	124.9	0	0
14	92.57	86.7	92.0	0	0
15.3	251.0	135.1	250.4	0	0
15.2	200.1	137.4	199.5	0	0
15.1	156.4	99.4	156.1	0	0
15	119.6	74.9	119.3	0	0
16.2	262.3	0	262.7	0	35 0
16.1	214.2	147.2	214.6	0	0
16	173.2	74.9	174.0	0	0
17.1	195.8	0	196.2	0	0
17	150.5	109.2	151.4	0	0
18	0	0	0	0	0
18.1	0	0	0	0	4982
18.2	0	0	0	0	8233

Again we assume that the average nonconforming lots frequency per period $\lambda = 1$, the distribution of the loss amount of one nonconforming lot is Normal distribution with mean equals 300, and standard deviation equals 10 such that $X \sim N(300, 100)$. Table 8 shows the optimal strategy of the supplier for each class $\vec{x}^*$, the expectation of all future costs V^*, the probability of non-declared nonconforming lot p^* and the transient time before enter the withdraw absorbing class N^*.

Table 9 presents the optimal strategy $\vec{x}^*$ for different nonconforming lots frequency and different normal distribution of loss amount. We find that for small mean of loss amount ($\mu < 1000$) and as the nonconforming lots frequency λ increase, the supplier tends to declare every nonconforming lots. But for large mean of loss amount ($\mu > 1000$) , the supplier should have large amount of critical points in last two classes, to keep him in the rating system. In the latter case, the average transient time is over 2000 periods for every classes of the rating system.

V. Conclusion

We describe how to compute the optimal strategy for quality rating system. The strategy determines the supplier whether or not to declare a nonconforming lot to his customer. The model contains regular and absorbing Markovian processs.

From column 8 of Table 3, the supplier, which has average one nonconforming lot per period, will spend most of his time in the lower classes under the optimal strategy. From column 5, it is very probable that the supplier will pay these nonconforming lots by himself.

The optimal strategy dependents on the nonconforming lot frequency λ. For every classes, the optimal retention limit x_i tends to zero for increasing λ. This means that if the average number of nonconforming lots of the supplier is more than five nonconforming lots per period, then he has to report every nonconforming lots to the customer.

The optimal strategy will dependents on the distribution of the loss amount of one nonconforming lot, and also dependents on the discount factor **q**. But the optimal strategies are not so sensitive with mean and standard deviation of X and **q**.

For absorbing Markovian model problem the optimal strategy is that the supplier will pay every nonconforming lots by himself or have large amount of critical points in last two classes.

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馬可夫評等系統之最適品質策略

陳文賢*

摘要

假設供應商與顧客之品質鏈使用評等系統。在此系統中一個供應商如果在一段時間內，有不良品給顧客，則該供應商會被降等，如果在該段時間內，沒有不良品給顧客，則該供應商會被升級。本論文利用隨機過程模式，假設馬可夫性質，利用反覆演算法，計算評等系統供應商之最適品質策略，該策略決定供應商何時該自行報銷不良品或宣告給顧客，以達到最適品質成本。

關鍵詞:品質評等系統、高斯消去法、馬可夫性質、最適化策略

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