ASSET SUBSTITUTION AND CREDIT REPUTATION EQUILIBRIUM

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Abstract

The purpose of this paper is to study the asset substitution problem in a credit market with repeated borrowing. In literature this agency problem between the borrower and the lender has been used to explain various financial phenomena. Applying the concept of sequential equilibrium, this study demonstrates that the role of reputation in a credit market can deal with the asset substitution problem without contracting cost. The nature of the equilibrium interest rate is explored and the welfare implication is discussed. Since the agency costs can be reduced in this equilibrium, the explanations based on asset substitution argument need to be examined more carefully.

Key words: asset substitution, risk shifting, agency cost, reputation, credit market

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I. Introduction

The asset substitution problem between a borrower and a lender has been studied extensively in the literature. (Jensen and Meckling, 1976; Galai and Masulis, 1976; Gavish and Kalay, 1983; Green, 1984; Green and Talmor, 1985). Because equity is an option with exercise price equal to the face value of the debt, the firm has an incentive to invest in a riskier project after the terms of the debt financing have been determined. In equilibrium the lender rationally anticipates this problem and writes the loan contract accordingly. The agency cost occurs if the riskier project is suboptimal to the firm.

The understanding of this substitution problem is important as several financial phenomena are associated with its existence. Jensen and Meckling (1976) provide a theory of optimal capital structure with this agency cost argument. Smith and Warner (1979) analyze the various debt covenants to control the possible asset substitution. Solutions of this problem are also related to debt maturity structure and call provision (Barnea, Haugen and Senbet, 1980). Recently, Campbell and Kracaw (1990) give the rational for headging diversifiable risk as it can reduce the agency cost from asset substitution. An optimal tax structure is proposed and a revenue neutral deposit insurance is specified to solve the risk substitution problem in banking firms (John, John and Senbet, 1991).

The purpose of this paper is to reexamine the asset substitution problem in a two-period framework. The model considers a representative firm which can borrow funds to invest in a project. However, after the loan is granted, the firm may substitute the original project with a riskier project. The lender has no knowledge about the riskiness of the substitute project. Only the firm knows its risk and must decide whether to substitute or not. In the second period the above situation repeats, but the lender rationally updates its belief about the riskness of the substitute project by observing the firm's past investment decision. The firm makes the investment decision taking this updated belief into account along with its private information and the future strategies. The firm's optimal strategy and the lender's rational beliefs are mutually consistent in the sense of the sequential equilibrium concept (Kreps and Wilson, 1982).

We show that there exists a credit reputation equilibrium in which agency cost from the asset substitution problem can be reduced. The lender's evaluation in the second period may be so bad that some firms "automatically" curtail their attempts for wealth transfer. But other firms do substitute because of the subsidized interest rate. In general low-risk firms will have to subsidize the high-risk firms. The subsidy is set to the level that the investment actions can be used to distinguish the borrowers. This reputational equilibrium may be superior to the non-reputational equilibrium in the sense of Pareto dominance.

The model demonstrates a self-enforcing mechanism in the market to deal with asset substitution problem without contracting cost. As a result, the significance of its impact on financial decisions need to be examined more carefully.

In section 2 the asset substitution problem is introduced. The two-period model is presented in section 3. Section 4 discusses the credit reputation equilibrium and the nature of the interest rate. Conclusions are provided in section 5. The proofs of the existence theorems are outlined in the appendix.

II. Asset Substitution Problem

We first consider a one-period model with a representative firm incorporated at the beginning and endowed with a riskless project K. This project requires an investment of \$1 and has payoff k at the end of the period. Assume that the firm has no investable resources but can borrow \$1 (and only \$1) from the credit market to invest in this project. If it does, the payoff k is then realized and the firm pays its debt and distributes the residuals to its equity holders.

Assume that both the borrowing firm and the lender are risk neutral. The credit market is perfectly competitive in the sense that loanable funds are available in perfectly elastic supply in the deposit market at cost r. The lenders earn zero expected profits in equilibrium due to perfect competition. Thus the expected payment for \$1 loan is r. We assume no taxes, no transaction costs or bankruptcy costs. The firm's liability is limited to its total value.

Suppose after the \$1 loan being granted, the firm has an opportunity to substitute project K with a risky project R. This project also requires \$1 investment but generates a random payoff X. We assume that X is normally distributed with mean u and standard deviation w, i.e., X~N(u,w). Standard deviation is used as the risk index in the model[1]. Thus an increase in w means an increase in the riskness of the project R.

The parameters specified above creat an asset substitution problem for our analysis. If the fixed amount of debt obligation is c, E[max{X-c,0}] is the firm's expected payoff from project R. Therefore, given a fixed-cost debt contract, the firm has the incentive to invest in a project as risky as possible[2]. We assume that $u < k$[3] and the risk of the project R is high enough so that $E[\max\{X-r,0\}] > k - r$. That is, even though the project R has the lower expected return u, the firm will take it if the lender anticipates the project K to be invested and charges r for repayment.[3]

In equilibrium the lender rationally expect this substitution of the risky project R for the riskless project K. The repayment c will be adjusted higher to satisfy $E[\min\{X,c\}] = r$ so that the lender has zero expected profit. The firm's expected payoff from project R would then be $E[\max\{X-c,0\}] = u-r$. We notice that $u-r < k-r$, where k-r is the firm's payoff from riskless project K. The firm suffers a loss of value, (k-r)-(u-r)=k-u. If the firm can credibly convince the lender that project K is its optimal choice and no substitution will be made, it can increase its market value.

[1]Since the risk of the project is the firm's private information in this model, any definition of risk that facilitates risk comparison in a mean preserving manner (Rothschild and Stiglitz(1970)) can serve our purpose.

[2]It is well established in the option pricing literature that $\partial (E[\max\{X-c,0\}]) / \partial w > 0$, here X~N(u,w) and c is a constant.

[3]If u>k or u=k, the firm surely chooses project R. The lender demands a repayment c for the debt such that $E[\min\{X,c\}] = r$. There is no risk shifting problem in this case.

III. The Two-period Model

Now consider a two-period model in which the above situation is repeated with parameters indexed by subscript 1 and 2. Assume that in the beginning of the period 1, both the firm and the lender are uncertain about the riskness of the project R1. The loan contract must be signed at this point and will expire at the end of this period. Let $F(w_1)$ be the prior distribution of riskness W1 with W1 in [a,b], where a is the minimum risk that can incur the agency problem, i.e., $E[\max\{X-r,0\}] > k - r$ if $X \sim N(u,a)$. A loan contract can be signed according to this prior distribution. Let c1 be the required repayment for this loan.

After receiving the funds for the investment, the firm learns the risk of the risky project, i.e., the standard deviation w1 of the distribution for X_1. It then decides either to invest in the risky project R_1 or to invest in the riskless project K_1. At the end of the period, the firm realizes the return and distributes the payoffs to the lender and/or the shareholders. The lender receives c_1 if K_1 is taken or $\text{Min}\{X_1,c_1\}$ if R_1 is taken. Similarly the firm receives k_1-c_1 or $\text{Max}\{X_1-c_1,0\}$ depending on its investment decision. The storage of the consumption goods and investable resources is not allowed.

The lender does not know what the realized risk w1 is, but it can find out the firm's investment decision at the end of the first period. We assume that a loan contract contingent on the investment decision is not enforceable in the first period[4].

In the second period, the riskless project K_2 is identical to K_1. The risky project R_2 is related to R_1 in the sense that higher risk of R_1 in the first period statistically implies higher risk of R_2 in the second period. Assume that $F(w_2|w_1)$ satisfies the first order stochastic dominance property, i.e., $F(w_2|w_1) \leqq F(w_2|w_1')$ if $w_1 \geqq w_1'$. For easier calculation, let[5]

[4]The purpose of this assumption is to create the agency problem in this simple model. In a real agency situation, we notice that things can be known but difficult or expensive to prove (in a legal sense) in order to enforce the contract.

$$F(w_2|w_1) = H(w_2) \quad \text{if } w_1 \geqq w^c,$$
$$F(w_2|w_1) = L(w_2) \quad \text{if } w_1 < w^c$$

where $H(w_2)$ dominates $L(w_2)$ in the sense of the first order stochastic dominance. Thus, if the firm observes w_1 larger than w^c which is a critical risk level known by the firm, it knows that project R_2 belongs to a higher risk type with distribution $H(w_2)$.

The lender does not know W_2. However, from the firm's project decision in the first period, the lender can modify his evaluation about the riskness of R_2. Denote $G(w_2|d_1)$ to be the lender's posterior evaluation for W_2, which depends on the firm's investment decision d1 in the first period.

Again a \$1 loan will be obtained to make the investment in the second period. Let $c_2(d_1)$be the required repayment for this contract. The firm then learns the risk of R_2 and makes the investment decision. At the end of the period the return is realized and the payoffs are distributed.

IV. The Credit Reputation Equilibrium

We first describe the strategies for the firm. Assume that two projects can be chosen randomly based on available information and the past decisions. Denote

$s_1(d_1|w_1)$ = probability of choosing d1 in $\{K_1,R_1\}$ in the first period given the observed risk w_1.

$s_2(d_2|w_1,d_1,w_2)$ = probability of choosing d_2 in $\{K_2,R_2\}$ in period 2 given observed w_1, w_2 and previous decision d_1.

The future payoff of the firm is affected by the decision d_1, and d_1 is determined considering the obligation of repayment c_1. This repayment is in turn a function of the firm's optimal strategy and the lender's beliefs. Thus we come to the problem of the in-

[5]The information structure of the two risk type partition is mainly for technical convenience. The proof for the existence of the reputation equilibrium is much easier and intuitively conceivable.

teraction between the lender's belief and the firm's strategy. We use the concept of sequential equilibrium to specify this relationship in which each is rational given the other.[6]

An equilibrium of the two period model described above is defined to be

(a) a set of strategies for the firm, $s_1(d_1|w_1)$ and $s_2(d_2|w_1,d_1,w_2)$, which define the probabilities of project choice given available information and the past decision,

(b) lender's belief $g(w_2|d_1)$ which gives its posterior belief about w_2 given the observed project choice by the firm,

such that

(1) lender's belief is consistent with Bayes rule in the sense of Kreps and Wilson (1982) given strategies s_1 and past decision d_1, i.e.,[7]

$$g(w_2|d_1) = \frac{\int_a^b s_1(d_1|w_1)f(w_2|w_1)f(w_1)\,\delta\, w_1}{\int_a^b s_1(d_1|w_1)f(w_1)\,\delta\, w_1}$$

(2) Lender has ex ante zero expected profit for each debt contract given firm's optimum strategy and its own beliefs,

(3) Firm's strategy in each period maximizes shareholder's welfare given lender's beliefs and its own future strategies.

Given this equilibrium concept, we can characterize two equilibria for the model. The first is that the firm always takes the risky project in each period, and the lender does not learn anything from the firm's decision in the first period. In this case reputation has no effect and agency cost prevails. However, in the second equilibrium, the lender rationally updates its evaluation on the riskness of the substitute project in the se-

[6]The definition used here is similar to the one developed by Harris and Raviv (1985).

[7]As noted by Harris and Raviv(1985), if the denominator of lender's posterior is zero given $s_1(d_1|w_1)$, we require that $g(w_2|d_1)$ be the limit of a sequence $\{g^n(w_2|d_1)\}$ where g^n is obtained using some strategy $s_1{}^n(d_1|w_1)$ such that $s_1{}^n(d_1|w_1)$ approaches $s_1(d_1|w_1)$ if n approaches infinity and $s_1{}^n(d_1|w_1)>0$.

cond period. The firm takes this effect into account and uses a strategy that does reveal information about the risk. The firm will not invest in the risky project although it is optimal in the first period. The strategy is self-enforcing so that the lender is credibly convinced of the firm's intention.

Theorem 1: A non-reputational equilibrium of the two period model is given by:

(a) The firm always takes the risky project independent of its private information, i.e.,

$S_1(R_1|w_1)=1$, $S_1(K_1|w_1)=0$, for all w_1,

$S_2(R_2|w_1,d_1,w_2)=1$, $S_2(K_2|w_1,d_1,w_2)=0$, for all w_1,d_1,w_2.

(b) The lender does not infer any information from firm's decision d_1, i.e.,

$G(w_2|d_1) = L(w_2)F(w^c) + H(w_2)[1-F(w^c)]$ for all d_1.

Proof: see appendix.

Theorem 2: There exists a fixed risk level w* in (a,b) such that a reputational equilibrium of the repeated game is given by

(a) $S_1(R_1|w_1) = 1$ if $w_1 \geqq w^*$,

$= 0$ if $w_1 < w^*$,

$S_1(K_1|w_1) = 1 - S_1(R_1|w_1)$

$S_2(R_2|w_1,d_1,w_2) = 1$ for all w_1,d_1,w_2,

$S_2(K_2|w_1,d_1,w_2) = 0$ for all w_1,d_1,w_2.

(b) $G(w_2|K_1) = \{F(w^c)/F(w^*)\}L(w_2) + \{1-[F(w^c)/F(w^*)]\}H(w_2)$

$G(w_2|R_1) = H(w_2)$

Proof: see appendix.

Theorem 2 states that the borrower will not take the risky project (no asset substitution problem) in period 1 if the realized project risk is smaller than a predetermined risk level w*. However, if the project risk is higher than w*, the borrower can't resist taking advantage of the agency situation by substituting riskier project for riskfree project. The lender rationally anticipates this situation and charges an interest rate taking all this into account. In equilibrium it earns zero-expected profit.

We see that the borrower's action depends on its own risk class. Some firms with the lower risk project want to maintain good reputation by choosing riskfree project. This action is not caused by the lender's monopoly power since we have a competitive credit market. Instead, the reputation established in the earlier period has its economic value in the future. The agency cost isreduced because the lender is convinced that these firms will not engage in the wealth transfer scheme even if the loan has been granted already.

In the second period, all the firms choose the risky project since there are no future benefits for the good behavior. However, the result in the reputational equilibrium is different from that of the normal backward induction. The firms are reclassified in the second period and charged differently for different risk class. This reclassification is based on the borrower's reputation in the earlier period. Thus the project choice in the first period is affected by this reclassification in the second period. If the benefits from a lower interest rate in the second period are larger than the benefits from the wealth transfer due to agency situation in the first period, the firm would choose the riskless project in the first period.

Intuitively, the high-risk type firms get a lower interest rate in the first period relative to its project risk. Thus a high-risk type firm won't mimic a low-risk type firm (by taking K_1) due to this interest subsidy. The cheating can get some future reputation benefits, but it is smaller than the subsidy.

Meanwhile, the low-risk type firms are charged a higher interest rate in the first period relative to its choice of the riskfree project. The low-risk type firms subsidize the high-risk type firms in the first period so that the firm type can be partially distinguished in the second period. Once the lender is able to group the different customers according to the credit record, the low-risk type firm benefits from its good reputation. The benefit comes from the lower interest rate (relative to its risk class) in the future which is the result of a better posterior belief about w_2 from the lender. The lender can rationally infer that firms taking risky project are high-risk type customers. In the second period, the interest rates are higher for them. The low-risk type firm therefore won't mimic the high-risk type firm.

Theorem 3: There exist values of the exogenous parameters such that the reputational equilibrium ex ante Pareto dominates the non-reputational equilibrium.

Since some borrowers automatically curtail the attempts of the wealth transfer from the lender, the social value is increased due to the decreased agency cost and the borrowers benefit all the surplus. Therefore the reputational equilibrium dominates the non-reputational equilibrium.

V. Conclusions

The asset substitution problem is studied in a credit market with repeated borrowing-lending relationship. Using the concept of sequential equilibria, we endogenize the reputation effect and prove the existence of the credit reputation equilibrium in which the agency cost can be reduced.

In this equilibrium, credit reputation plays an important role in the negotiation of the loan contract. The low-risk type firms value good reputation for future benefits and won't take advantage of the agency relationship. The lender thus can charge a lower interest rate. The social welfare increases because of the reduced agency cost.

These decisions in the credit reputation equilibrium are self-enforcing without any contracting cost. Thus the asset substitution is less serious given a continuous credit relationship. Since various financial phenomena are explained due to this problem, its impact need to be re-examined more carefully.

We have assumed that the firm cannot use the internal funds for future investment. To the extent that the firm has to borrow from the credit market in the future, the model can be relaxed to allow the internal financing. The nature of the results does not change. However, if the borrowing amount can serve as another signal to the credit market, this greatly complicates the model and its impact remained to be solved. Also, more insights of reputation effect can be explored if we can have longer periods in the model. These topics are left for further study.

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Appendix : Outline of the proofs

Theorem 1:

Since the denominator of the posterior is zero, we utilize the discussion in footnote 7. For $n > 0$, define

$$S_1^n(d_1|w_1) = 1 - 1/n \quad \text{if } d_1 = R_1,$$
$$= 1/n \quad \text{if } d_1 = K_1.$$

$$g^n(w_2|d_1) = \frac{\int S_1^n(d_1|w_1)f(w_2|w_1)f(w_1)\,\delta\, w_1}{\int S_1^n(d_1|w_1)f(w_1)\,\delta\, w_1}$$

Therefore, $g^n(w_2|R_1) = g^n(w_2|K_1)$

$$= L(w_2)F(w^c)+H(w_2)(1-F(w^c)).$$

We have $EV(w_1,R_1,w_2) = EV(w_1,K_1,w_2)$

Given the agency assumption, it can be checked that

$y^* = z^* = 1$, for all w_1 and w_2. Q.E.D.

Theorem 2:

To prove the existence of the reputational equilibrium, We need the following definitions and lemmas:

Define $S_1^v(R_1|w_1) = 1 \quad \text{if } w_1 \geqq v,$

$= 0 \quad \text{if } w_1 < v, \quad v \text{ in } (a,b),$

then $S_1^v(K_1|w_1) = 1 - S_1^v(R_1|w_1)$.

$S_1^v(d_1|w_1)$ can be viewed as the lender's conjecture about the borrower's strategy which is characterized by the critical value of risk level v. In equilibrium $S_1^v(d_1|w_1)$

should be the same as $S_1(d_1|w_1)$ in theorem 2, so the interest rates are correctly charged to earn zero-expected profit.

Let $G^v(w_2|d_1)$ be the lender's posterior belief if the borrower uses strategy $S_1^v(d_1|w_1)$ as conjectured. We have

$$G^v(w_2|d_1) = \frac{\int_a^b S_1^v(d_1|w_1)f(w_2|w_1)f(w_1)\,\delta w_1}{\int_a^b S_1^v(d_1|w_1)f(w_1)\,\delta w_1}$$

Let $c_2^v(d_1)$ be the interest payment for the second period if $S_1^v(d_1|w_1)$ is the borrower's strategy and d_1 is observed. Then $c_2^v(d_1)$ is the solution of

$$E_{X2}E_{w2}[\mathrm{Min}\{X_2,c_2^v(d_1)\}|G^v(w_2|d_1)] = r.$$

Let $V^v(d_1,w_2)$ be the borrower's optimum value in the second period if $S_1^v(d_1|w_1)$ is the strategy and d_1 is the project choice.

$$V^v(d_1,w_1) = E_{X2}E_{w2}[\mathrm{Max}\{X_2-c_2^v(d_1),0\}|F(w_2|w_1)].$$

Define $Q(v,w_1)$ to be the reputation benefit if $S_1^v(d_1|w_1)$ is the strategy and w_1 is realized, i.e.,

$$Q(v,w_1) = V^v(K_1,w_2)-V^v(R_1,w_2).$$

$Q(v,w_1)$ is the difference in the optimum value for the second period between good reputation (investing in K_1) and bad reputation (investing in R_1).

Let c_1^v be the interest payment in the first period if $S_1^v(d_1|w_1)$ is the borrower's strategy. Then c_1^v is the solution of

$E_{w1}\{S_1^v(R_1|w_1)E_{X1}[Min\{x_1,c_1^v\}]+S_1^v(K_1|w_1)c_1^v|F(w_1)\} = r.$

Define $B(v,w_1)$ to be the benefit from investing in R_1 instead of K_1 under strategy $S_1^v(d_1|w_1)$ given c as the interest payment in the first period, i.e.,

$B(v,w_1) = E_{X1}[Max\{X_1\text{-}c_1^v,0\}]\text{-}(k\text{-}c_1^v).$

$B(v,w_1)$ is the extra wealth which the borrower can transfer from the lender in the first period due to the agency relationship.

Lemma 1: If $v \geqq w^c$, $\delta Q(v,w_1)/\delta v < 0$.
If $v < w^c$, $\delta Q(v,w_1)/\delta v > 0$.
If $w_1 \neq w^c$, $\delta Q(v,w_1)/\delta w_1 = 0$.

Pf: $Q(v,w_1) = EE[Max\{x_2\text{-}c_2^v(K_1),0\}|w_2]\text{-}EE[Max\{x_2\text{-}c_2^v(R_1),0\}|w_2]$

$$\frac{\delta Q(v,w_1)}{\delta v} = \frac{\delta c_2^v(K_1)}{\delta v} + \frac{\delta Q}{\delta c_2^v(K_1)} + \frac{\delta c_2^v(R_1)}{\delta v} + \frac{\delta Q}{\delta c_2^v(R_1)}$$

If $v \geqq w^c$, from first order stochastic dominance property,

$\delta c_2^v(K_1)/\delta v>0,\ \delta Q/\delta c_2^v(K_1)<0,\ \delta c_2^v(R_1)/\delta v=0.$

Therefore, $\delta Q(v,w_1)/\delta v<0.$

If $v < w^c$, $\delta c_2^v(K_1)/\delta v=0,\ \delta c_2^v(R_1)/\delta v<0,\ \delta Q/\delta c_2^v(R_1)>0.$

Therefore, $\delta Q(v,w_1)/\delta v>0.$

Since $\delta Q(v,w_1)/\delta w_1= [\delta f(w_2|w_1)/\delta w_1](\delta Q/\delta f).$

If $w_1 \neq w^c$, $\delta f(w_2|w_1)/\delta w_1=0$. So $\delta Q(v,w_1)/\delta w_1=0.$ Q.E.D.

Lemma 2: $\delta B(v,w_1)/\delta v < 0,\ \delta B(v,w_1)/\delta w_1 >0.$

Pf: $B(v,w_1) = E[Max\{x_1,c_1^v\}]\text{-}k$

$\delta B(v,w_1)/\delta v = (\delta c_1^v/\delta v)[\delta B(v,w_1)/\delta c_1^v]$.

We have $\delta B(v,w_1)/\delta c_1^v > 0$.

From the constraint

$E\{[S_1^v(R_1|w_1)E[\min\{x_1,c_1^v\}]+[1-S_1^v(R_1|w_1)]c_1^v]|w_1\}=r$

$E\{S_1^v(R_1|w_1)E[\min\{x_1,c_1^v\}]|w_1\}+c_1^v = r$

Total differentiate it with respect to v,

$\delta c_1^v/\delta v = [(x_1-c_1^v)n(x_1)f(w_1)d_{x1}]/[1-(1-F(v))N(c_1^v)] <0$

where N is the distribution function of X_1.

Therefore, $\delta B(v,w_1)/\delta v <0$.

From footnote 2,

$\delta B(v,w_1)/\delta w_1 = \delta E[\text{Max}\{x_1,c_1^v\}]/\delta w_1 > 0$. Q.E.D.

Lemma 3: If $Q(w^c,w^c) > B(w^c,w^c)$, then there exists w* in (a,b) such that $Q(w^*,w^*) = B(w^*,w^*)$.

Pf: Since $Q(w^c,w^c) > B(w^c,w^c)$ and $0 = Q(b,w^c) < B(b,w^c)$,

there exists v_1^*, $w^c<v_1^* < b$, s.t. $Q(v_1^*,w^c) = B(v_1^*,w^c)$.

From Lemma 1 and Lemma 2, there exists v_2^* and w_1^*,

where $w^c \leqq v_2^* < b$, $w^c < w_1^* \leqq b$,

such that $Q(v_2^*,w_1^*) = B(v_2^*,w_1^*)$.

By the fixed point theorem,

there exist w*, $w^c < w^* < b$, $Q(w^*,w^*) = B(w^*,w^*)$. Q.E.D.

Theorem 2:

Let $y(w_1,d_1,w_2)$ be the probability of choosing R_2.

In the second period, we have

$$\underset{y}{\text{Max}}\ y\, E[\text{Max}\{x_2-c_2(d_1),0\}]+(1-y)\{k-c_2(d_1)\}$$

where $c_2(d_1)$ satisfies

$E\{y^*E[Min\{x_2,c_2(d_1)\}]+(1-y^*)c_2(d_1)|G(w_2|d_1)\}=r$

where y* is the optimal solution.

After simplification, we have

$$\underset{y}{Max}\ y\{E[Max\{x_2-c_2(d_1),0\}-[k-c_2(d_1)]]\}+k-c_2(d_1)$$

Define $J(c_2(d_1))=E[Max\{x_2-c_2(d_1),0\}-[k-c_2(d_1)]$

Therefore, $\underset{y}{Max}\ y\{J(c_2(d_1))\}+k-c_2(d_1)$

From the assumption, $J(r) > 0$.

From the constraint $J(c_2(d_1)) > J(r)$.

Since $\delta J(c_2)/\delta c_2>0$, so $J(c_2(d_1)) > 0$ for all $c_2(d_1)$.

We have $y^* = 1$ for all w_1, d_1, w_2.

And $c_2(d_1)$ can satisfy

$$E_{X2}E_{w2}[Min\{X_2,c_2(d_1)\}|G(w_2|d_1)] = r.$$

Define $V(w_1,d_1,W_2)$ to be the maximum of the object function in the second period given action d_1 in the first period,

Let $z(w_1)$ be the probability of choosing R_1.

In the first period, we have

$$\underset{z}{Max}\ z\{E[Max\{x_1-c_1,0\}]+E[V(w_1,R_1,W_2)]\} + (1-z)\{(k-c_1)+E[V(w_1,K_1,W_2)]\}$$

s.t. $E\{z^*E[Min\{x_1,c_1\}]+(1-z^*)c_1|F(w_1)\} = r.$

where z* is the optimal solution.

Simplify the object function, we get

$$\underset{v}{Max}\ z(v)\{B(v,w_1)-Q(v,w_1)\}+(k-c_1)+E[V(w_1,K_1,W_2)]$$

where $z(v) = S_1^v(R_1|w_1)$.

Since there exists w* such that

$B(w^*,w^*) = Q(w^*,w^*)$ by Lemma 3.

If $v=w^*$ and the realized $w_1 w^*$, by Lemma 1 and Lemma 2,

$B(w^*,w_1) > Q(w^*,w_1)$. We get $z^*= 1$ in this case.

Similarly, if $w_1 < w^*$, we have $z^*= 0$.

So, $S_1(R_1|w_1) = 1$, if $w_1 \geqq w^*$,

$= 0$, if $w_1 < w^*$.

Using this strategy and the Bayes rule, it can be checked

that $G(w_2|K_1) = \{F(w^c)/F(w^*)\}L(w_2)+\{1-[F(w^c)/F(w^*)]\}H(w_2)$

$G(w_2|R_1) = H(w_2)$ Q.E.D.

信用市場中資產替代與企業信譽均衡關係之研究

王克陸

摘 要

本文之目的在探討多時期情況下之資產替代--企業轉移借款於風險較大之投資計劃--問題. 資產替代曾用於解釋資本結構, 多種債務契約條款, 可分散風險之風險防禦, 以及存款保險之保費規劃等等. 運用序列均衡之觀念, 本文証明了企業信譽在重複借貸均衡關係中解決資產替代問題的角色, 並探討其中利率的本質. 適當使用信用紀錄, 可降低資產替代問題衍生之代理成本. 同時, 文獻中一些依據資產替代問題所作之解釋, 需要進一步的驗證.

關鍵詞: 資產替代, 風險轉移, 代理成本, 企業信譽, 信用市場