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Ambiguity Increases and Insurance Deductibles

模糊增加與保險自負額

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Abstract

This paper investigates the impact of an ambiguity increase on the optimal insurance deductible for a risk- and ambiguity-averse individual under the uncertainty of a loss distribution. A deductible is an important insurance contract design in both theory and practice. Previous studies have reported preference-based results in the context of coinsurance, albeit with limited applications. In this paper, we prove a straight deductible is optimal under an α-maxmin model. In the context of the straight deductible, we assume that the cumulative loss probability at an initial optimal deductible is preserved after an ambiguity increase. We show that, for a loss below the initial optimal deductible, the optimal deductible remains unchanged when possible distributions are unaffected by the ambiguity increase. Allowing for a distinct center in the belief set while keeping the others unchanged, we prove that, when the worst distribution is unaffected, but the best distribution deteriorates in terms of first-order stochastic dominance, the optimal deductible becomes lower after the ambiguity increase. If the cumulative loss probability is not preserved, the optimal deductible decreases when, at the initial optimal deductible, the odds of obtaining partial indemnity relative to no indemnity become larger under the loss distribution distorted by ambiguity aversion.

【Keywords】ambiguity increase, ambiguity aversion, optimal insurance coverage, deductible, α-maxmin model

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摘 要

本文研究當損失分配不確定下，模糊增加對風險及模糊趨避者之最適保險自負額之影響。自負額是保險理論與實務重要的契約設計。之前研究針對共同保險議題所得結果為偏好相依，惟其應用困難。本文運用 α-maxmin 模型證明固定自負額為最適契約；接著假設累積損失機率在原自負額下不變，發現當低於原自負額之損失之可能分配在模糊增加後仍不變，最適選擇不變。若其他設定不變但允許決策者想法集合之中心改變，則發現當最差分配不受影響，但最佳分配以一階隨機優越的概念變差時，最適自負額在模糊增加後變低。若無累積損失機率不變之假設，在原自負額及因模糊趨避扭曲之損失分配下，當獲得部份賠償相較於零賠償的機會在模糊增加後變大時，最適自負額變低。

【關鍵字】模糊增加、模糊趨避、最適保險保障、自負額、α-maxmin 模型

1. Introduction

From firm managers to individual consumers, people usually make decisions under uncertainty in a changing environment. Uncertainty can come from different sources. When making a decision, one may not know the outcome before it is realized, but may know the probabilities of different outcomes (the outcome distribution). In this case, one faces a risk, as the outcome is uncertain. In other cases, in addition to the uncertainty of the outcome, the outcome distribution may also be uncertain. Then, the individual faces ambiguity in decision-making.

Ambiguity, first defined by Knight (1921), commonly refers to a situation where probabilities are unknown or not perfectly known. Uncertainty is not limited to the outcome distribution (probabilities). For example, it can relate to the distribution of the information affecting a decision, such as the default probability of a counterparty involved in a contract (e.g., Peter and Ying, 2020). In this case, the ambiguity concerns the uncertainty of the insurer's nonperformance probability. Another example is a parameter that determines the outcome distribution (e.g., Illeditsch, Ganguli, and Condie, 2021), where the ambiguity concerns the uncertainty of the correlation between a value distribution of a risky asset and the sign indicating its value. In the face of ambiguity, people tend to behave differently from a subjective-expected-utility (SEU) maximizer. For example, most people choose a prospect with certain payoff probabilities instead of one with uncertain payoff probabilities. This preference is called ambiguity aversion (Ellsberg, 1961). Recently, the preference of partial ambiguity aversion has been proposed and found from the experiment (Klingebiel and Zhu, 2023). This preference describes that the attitude toward ambiguity does not consistently exhibit over all degree of ambiguity but will switch from seeking to aversion toward ambiguity at a threshold of degree of ambiguity.

The influence of ambiguity aversion on decision-making under ambiguity has been documented in the literature on insurance markets.[1] One of the noteworthy issues is the

1 In addition to the insurance markets, some recent papers have also found that ambiguity aversion can explain firms' decision of leverage (e.g., Izhakian, Yermack, and Zender, 2022) and the positive relationship between risk and return in stock markets (e.g., Ghazi, Schneider, and Strauss, 2025).

decision on optimal insurance coverage or demand for (self-)insurance.[2] Insurance is an effective tool for transferring risks, and choosing the optimal insurance coverage is a common decision in daily life. When the risk is ambiguous, the optimal insurance contract under ambiguity aversion is in a deductible form (e.g., Alary et al., 2013; Gollier, 2014; Birghila et al., 2023). Furthermore, two streams of research have shown how the optimal insurance coverage changes with ambiguity aversion and ambiguity increases. One stream of research shows that greater ambiguity aversion increases the optimal insurance coverage in both the loss and no-loss states of nature (e.g., Snow, 2011; Alary et al., 2013). In multiple loss states, depending on how ambiguity is distributed, some papers report consistent results under certain conditions (e.g., Alary et al., 2013), while other papers report the opposite result (e.g., Gollier, 2011, 2014). The other stream of research shows that greater ambiguity increases the optimal insurance coverage under ambiguity aversion (e.g., Jewitt and Mukerji, 2017; Huang and Tzeng, 2018; Huang, 2025).

In this paper, we focus on the second stream of research, particularly on determining the conditions under which greater ambiguity leads to higher optimal insurance coverage. Based on the choices of ambiguity-averse individuals, Jewitt and Mukerji (2017) define two notions of one act being more ambiguous than another act. Specifically, one notion is defined by the preference of an ambiguity-averse individual relative to that of an ambiguity-neutral individual. The other notion is defined by the compensation for giving up one act for the other act required by a more ambiguity-averse individual compared to a less ambiguity-averse individual. When these notions are formulated via ambiguity models such as an α-maxmin model (Ghirardato, Maccheroni, and Marinacci, 2004), the determining conditions depend on utility functions, which are preference-related. Huang and Tzeng (2018) define an Nth-degree ambiguity increase as a distribution change in the sense of an Nth-degree increase in risk (Ekern, 1980). This preserves the mean under a smooth ambiguity aversion model (Klibanoff, Marinacci, and Mukerji, 2005) and broadens

2 The effect of ambiguity aversion has also been shown on equilibria in insurance markets with asymmetric information (e.g., Koufopoulos and Kozhan, 2014, 2016; Zheng, Wang, and Li, 2016), the optimal design for insurance contracts (e.g., Anwar and Zheng, 2012; Alary, Gollier, and Treich, 2013; Gollier, 2014; Birghila, Boonen, and Ghossoub, 2023), and insurance pricing (e.g., Cabantous, 2007; Cabantous, Hilton, Kunreuther, and Michel-Kerjan, 2011; Huang, Huang, and Tzeng, 2013; Amarante, Ghossoub, and Phelps, 2015; Dietz and Walker, 2019; Dietz and Niehörster, 2021).

the support under the α-maxmin model. Their determining conditions are defined based on risk and/or ambiguity preferences. These preference-related conditions are difficult to use in empirical verification and applications. Conversely, Huang (2025) provides the non-preference-based determining conditions defined as changes in the possible loss distributions under the α-maxmin model for greater ambiguity, defined as a larger set of the individual's beliefs. Nevertheless, the above papers all study the comparative statics of ambiguity under the problem of optimal coinsurance (optimal portfolio choices, equivalently) rather than deductibles.

Deductibles are an important aspect of insurance contracts. For example, they can be used to reduce moral hazard, as is common in automobile and health insurance. The literature indicates that deductibles are the optimal insurance design for risk and ambiguity-averse individuals through the smooth ambiguity aversion model (e.g., Alary et al., 2013; Gollier, 2014). Alary et al. (2013) prove that the straight deductible insurance is optimal when ambiguity concentrates in the no-loss state of nature. Meanwhile, Gollier (2014) shows that in multiple states of nature, the optimal insurance design exhibits different types of deductibles depending on the ambiguity structure. When formulating the decision-making with a maxmin expected utility (Gilboa and Schmeidler, 1989), Birghila et al. (2023) demonstrate that the deductible is optimal when the insurer is risk neutral. Moreover, under the α-maxmin model, Zhang and Li (2021) show that the optimal reinsurance contract is in an excess-of-loss form in certain cases.[3] Overall, to the best of our knowledge, two areas remain unexamined: whether a straight deductible under the α-maxmin model is still optimal for the insurance contract,[4] and how the optimal deductible responds to an ambiguity increase. The challenges associated with preference-based determining conditions and the importance of deductibles motivate us to study preference-free determining conditions for optimal deductibles under ambiguity.

This paper aims to investigate how the optimal insurance coverage of deductibles changes and when it increases with an ambiguity increase for a risk- and ambiguity-

3 An excess-of-loss form can be viewed as a form of deductible used in reinsurance contracts. Under a reinsurance contract with an excess-of-loss form, the reinsurer provides indemnity to the insurer when the insurer's claim payments (with respect to the ceded risks) exceed a certain amount or percentage.

4 We thank an anonymous reviewer for raising this point.

averse individual. Ambiguity comes from the uncertainty of the loss distribution, which is determined by the individual's beliefs. We study two types of ambiguity increases: specific and general. The former refers to an ambiguity increase that broadens the set of beliefs and preserves a cumulative loss probability at the initial optimal deductible level. The latter is an ambiguity increase that broadens the set of beliefs without preserving the requirement of the specific ambiguity increase. We focus on the risk and ambiguity aversion of the individual and assume that an insurer is risk and ambiguity neutral.[5] Since the insurer is ambiguity neutral, the loss distribution in premium pricing is not affected by the ambiguity increase. Finally, the optimal decisions are formulated by the α-maxmin model. Commonly employed in the literature, the α-maxmin model and its special case, the maxmin expected utility, offer several advantages: (1) separating the characterization of ambiguity from ambiguity preferences; (2) facilitating comparisons with results under (subjective) expected utility, and (3) accommodating a broader class of model uncertainty in decision-making (Birghila et al., 2023).[6,7]

We first provide a basis for our comparative statics on ambiguity by proving the optimality of a straight deductible. Then, we obtain necessary and sufficient conditions defined on the possible loss distributions, which describe how the risk- and ambiguity-averse individual reacts to the two types of ambiguity increases. After the specific ambiguity increase, the individual keeps the same deductible level as long as the possible loss distributions below the initial optimal deductible level remain unaffected. We also find that the optimal deductible becomes lower after a specific ambiguity increase allows for a distinct center in the belief set (i.e., a nonspecific ambiguity increase). This occurs

5 Throughout the paper, ambiguity aversion refers to (full) ambiguity aversion. We do not consider the partial ambiguity aversion (Klingebiel and Zhu, 2023).

6 These two ambiguity models are commonly used in the literature to formulate static insurance decisions (e.g., Anwar and Zheng, 2012; Koufopoulos and Kozhan, 2014, 2016; Amarante et al., 2015; Huang and Tzeng, 2018; Dietz and Walker, 2019; Dietz and Niehörster, 2021) and portfolio choices (e.g., Fei, 2009; Bossaerts, Ghirardato, Guarnaschelli, and Zame, 2010; Epstein and Schneider, 2010; Jewitt and Mukerji, 2017; Illeditsch et al., 2021).

7 Other ambiguity models have been proposed in the literature, such as the Choquet expected utility (Schmeidler, 1989), the recursive multiple-priors utility (Epstein and Schneider, 2003), the smooth ambiguity aversion model, and the local and global multiple-prior representations of ambiguity (Ghirardato and Siniscalchi, 2012).

when the worst loss distribution is unaffected but the best loss distribution below the initial optimal deductible deteriorates in the sense of first-order stochastic dominance (FSD). The conditions under the nonspecific ambiguity increase are analogous to, yet more than, those associated with risk changes in the absence of ambiguity, as reported by Powers and Tzeng (2001). On the other hand, after a general ambiguity increase, the individual chooses a lower deductible level when the odds of obtaining partial indemnity relative to no indemnity increases under the loss distribution distorted by ambiguity aversion.

This paper makes several contributions. First, regarding decision-making under ambiguity that can be characterized by the α-maxmin model, we believe this is the first paper that examines the optimality of a straight deductible and studies the comparative statics of ambiguity on the optimal deductible under ambiguity aversion. Second, our comparative statics of ambiguity can be viewed as an extension of the work by Birghila et al. (2023), who study only the optimal insurance contract with a special case of our model (the maxmin expected utility). In addition, our paper studies a dual problem of the work of Alary et al. (2013) and Gollier (2014), who study the optimal deductible under ambiguity aversion versus ambiguity neutrality for certain ambiguity structures with a different model. Moreover, our determining conditions under the specific and nonspecific ambiguity increases are preference-free and easily applicable to future empirical studies. Finally, our results provide insurers with a reference for analyzing the insurance purchasing behavior of risk- and ambiguity-averse individuals facing ambiguous risks.

The remainder of this paper is organized as follows. Section 2 introduces the model settings and proves the optimality of deductibles. Section 3 studies the effects of specific and general ambiguity increases on the optimal deductible. Section 4 concludes the paper. All the proofs are presented in the appendices.

2. Model Settings and the Optimal Insurance Contract

In this section, we describe the settings and assumptions made for the analysis. Then, we present our model. Finally, we examine the optimal insurance contract in the presence of ambiguity.

2.1 Model Settings

Suppose that there is a risk- and ambiguity-averse individual with initial wealth w and a potential loss $x \in [0,L]$. The risk is ambiguous in the way that the loss distribution $G(x;\pi)$ is determined by the individual's belief π. A set of π is $\Pi_F = \{\pi \mid \pi_F \leq \pi \leq \overline{\pi}_F\}$. In other words, ambiguity is characterized by Π_F. In this paper, we assume a simple ambiguity structure where the ambiguity exists for all $x, \in [0,L]$ to focus on the comparative statics of an ambiguity increase.[8]

Let Λ denote a set of all loss distributions and $\Delta_F \subseteq \Lambda$ denote a set of the loss distributions when $x \in \Pi_F$. Note that, when Π_F includes only a singleton, the loss distribution is certain. In this case, the decision maker faces risk instead of ambiguity since the uncertainty exists only for the loss. Accordingly, the decision maker behaves as an SEU maximizer, which is consistent with ambiguity neutrality.[9] For decision-making under ambiguity neutrality, we make the following assumption:

Assumption 1: Under ambiguity neutrality (in the absence of ambiguity), the loss distribution is $G(x;\pi^*) \in \Delta_F$, where $\pi^* \in \Pi_F$.

To transfer risk, the individual intends to buy an insurance contract from a risk- and ambiguity-neutral insurer. To focus on the purchasing behavior in response to an ambiguity increase, we assume that the ambiguity affects the individual only. The insurer is assumed to be risk- and ambiguity-neutral, consistent with several previous studies on comparative statics (e.g., Snow, 2011; Alary et al., 2013; Gollier, 2014; Jewitt and Mukerji, 2017; Huang and Tzeng, 2018; Peter and Ying, 2020; Birghila et al., 2023; Huang, 2025). An insurer who prices contracts mainly based on actuarial expertise, experience, and large data can be expected to be risk and ambiguity neutral in premium pricing decision-making.[10] Furthermore, when the insurer is risk averse, the deductible may not be optimal,

8 Gollier (2014) investigates the optimal insurance contract under different ambiguity structures, such as the ambiguity occurring at the loss below or above the initial optimal deductible level.

9 Without loss of generality, the SEU can represent an ambiguity-neutral preference (e.g., Jewitt and Mukerji, 2017).

10 It is noted that insurers can have different risk and ambiguity attitudes, such as exhibiting ambiguity aversion in the face of ambiguity by charging a higher premium (e.g., Cabantous, 2007; Cabantous et al., 2011). Some researchers study the optimal insurance coverage when the insurer is risk neutral

as proven by Birghila et al. (2023) under the maxmin expected utility. Under the insurance contract, the insurer will pay an indemnity denoted by $I(x) \geq 0$ to the individual when the loss occurs in the future. For simplicity, we assume the indemnity to be non-negative for all $x \in [0,L]$, as in Alary et al. (2013) and Gollier (2014), without being further limited by the no-sabotage condition (Birghila et al., 2023).[11]

The insurance premium paid by the individual (P) is assumed to be actuarially priced as an expected loss covered with a loading factor $\tau > 0$, which is expressed as

$$P = (1+\tau)\int_0^L I(x)\, dG(x;\pi^*). \tag{1}$$

Since the insurer is ambiguity neutral, under Assumption 1, the loss distribution used in the premium pricing is $G(x;\pi^*)$.

Under the α-maxmin model, previous works have defined the property of central symmetry for the set of the probability measure of state space (Rogers and Ryan, 2012), the set of belief associated with the preferences (Jewitt and Mukerji, 2017; Dietz and Walker, 2019), and the set of the net wealth distributions associated with the beliefs (Huang, 2025). For Δ_F, we similarly define the central symmetry as follows.

Definition 1: The set $\Delta_F \subseteq \Lambda$ is centrally symmetric if there exists a center $G(x;\pi^*) \in \Delta_F$ where $\pi^* \in \Pi_F$ such that, for any $G \in \Lambda$, $G \in \Lambda_F$ if and only if $G(x;\pi^*)$ - $[G(x;\pi)$ - $G(x;\pi^*)] \in \Delta_F$ for all $x \in [0,L]$.

We illustrate the meaning of Definition 1 with Panel A of Figure 1.[12] Let us consider a numerical example with the discrete loss x taking a value in the set of $\{0,1,2,3\}$. The set Δ_F has a center $G(x;\pi^*)$, a lower bound $G(x;\underline{\pi}_F)$, and an upper bound $G(x;\overline{\pi}_F)$. The dark dotted line draws $G(x;\pi^*)$, under which the cumulative loss probabilities of the possible loss

and ambiguity averse (e.g., Amarante et al., 2015; Dietz and Walker, 2019; Dietz and Niehörster, 2021) or risk averse (risk neutral) and ambiguity neutral (e.g., Birghila et al., 2023). We appreciate an anonymous reviewer's suggestion to justify the assumptions regarding the insurer's risk and ambiguity preferences.

11 The no-sabotage condition, which requires the retention to be comonotonic with the indemnity, is imposed on the indemnity function in the literature for the non-expected utility to avoid the ex post moral hazard. Birghila et al. (2023) demonstrate that the no-sabotage condition affects the shape of the optimal indemnity function.

12 We thank an anonymous reviewer for this suggestion.

values are 0.25, 0.5, 0.75, and 1, respectively. The dark dashed line draws $G(x;\underline{\pi}_F)$, under which the cumulative loss probabilities are 0.2, 0.4, 0.6, and 1, respectively. Furthermore, the dark solid line draws $G(x;\overline{\pi}_F)$, under which the cumulative loss probabilities are 0.3, 0.6, 0.9, and 1, respectively. Now, suppose that there is a loss distribution, $G(x;\pi_1) \in \Lambda$ (the light solid line), under which the cumulative loss probabilities are 0.25, 0.45, 0.8, and 1, respectively. There is another distribution, $G(x;\pi_1^*) = G(x;\pi^*) - [G(x;\pi_1) - G(x;\pi^*)]$ (the light dashed line), which is constructed by $G(x;\pi_1)$ when $\alpha = 0$ and, for $x > 0$, symmetrically expanding the center $G(x;\pi^*)$ in the opposite direction with the difference between $G(x;\pi_1)$ and $G(x;\pi^*)$ (see Panel A). The set Δ_F is central symmetric because both $G(x;\pi_1)$ and $G(x;\pi_1^*)$ belong to $\in \Delta_F$ for all $x \in [0,3]$ with the center $G(x;\pi^*)$, which satisfies Definition 1.

A counterexample is provided in Panel B of Figure 1. Here, we consider a loss distribution $G(x;\pi_2) \in \Delta$ (the light solid line), under which the cumulative loss probabilities are 0.4, 0.7, 0.9, and 1, respectively, and the distribution $G(x;\pi_2^*)$ (the light dashed line) constructed in a way similar to $G(x;\pi_1^*)$. Except for the loss $x = 3$, both $G(x;\pi_2^*)$ and $G(x;\pi_2)$ are outside of Δ_F for all x. In this case, Definition 1 is not satisfied, and thus, Δ_F is not central symmetric.

We adopt the α-maxmin model to characterize the individual's decision problem. The α-maxmin model takes the form of the α-weighted average of the maxmin expected utility and the maxmax expected utility, where $\alpha \in [0,1]$ describes the decision-maker's ambiguity preference. Let z denote the net wealth after loss and $v(I(x);u,z,\alpha,G,\pi)$ denote utility under the insurance contract with the indemnity function $I(x)$. To proceed with our analysis, we make the following assumptions for Δ_F and v, consistent with the study of the optimal coinsurance resulting from an ambiguity increase (Huang, 2025).

Assumption 2: The set $\Delta_F \subseteq \Lambda$ is compact, convex, and centrally symmetric (Definition 1) with a center $G(x;\pi^*) \in \Delta_F$, where $\pi^* \in \Pi_F$.

Assumption 3: $v(I(x);u,z,\alpha,G,\pi)$ is nondecreasing in π.

We justify these assumptions as follows. As noted by Jewitt and Mukerji (2017), Rogers and Ryan (2012) prove that, for the α-maxmin preferences under Assumption 2, ambiguity neutrality is described by $\alpha = \frac{1}{2}$. Since $\pi \in \Pi_F$, the center $G(x;\pi^*) \in \Delta_F$ is $\frac{1}{2}G(x;\underline{\pi}_F) + \frac{1}{2}G(x;\overline{\pi}_F)$. We make this assumption to ensure that the decision-making

Panel A: A Central Symmetric Set of Loss Distribution (Δ_F)

Panel B: A Non-Central Symmetric Set of Loss Distribution (Δ_F)

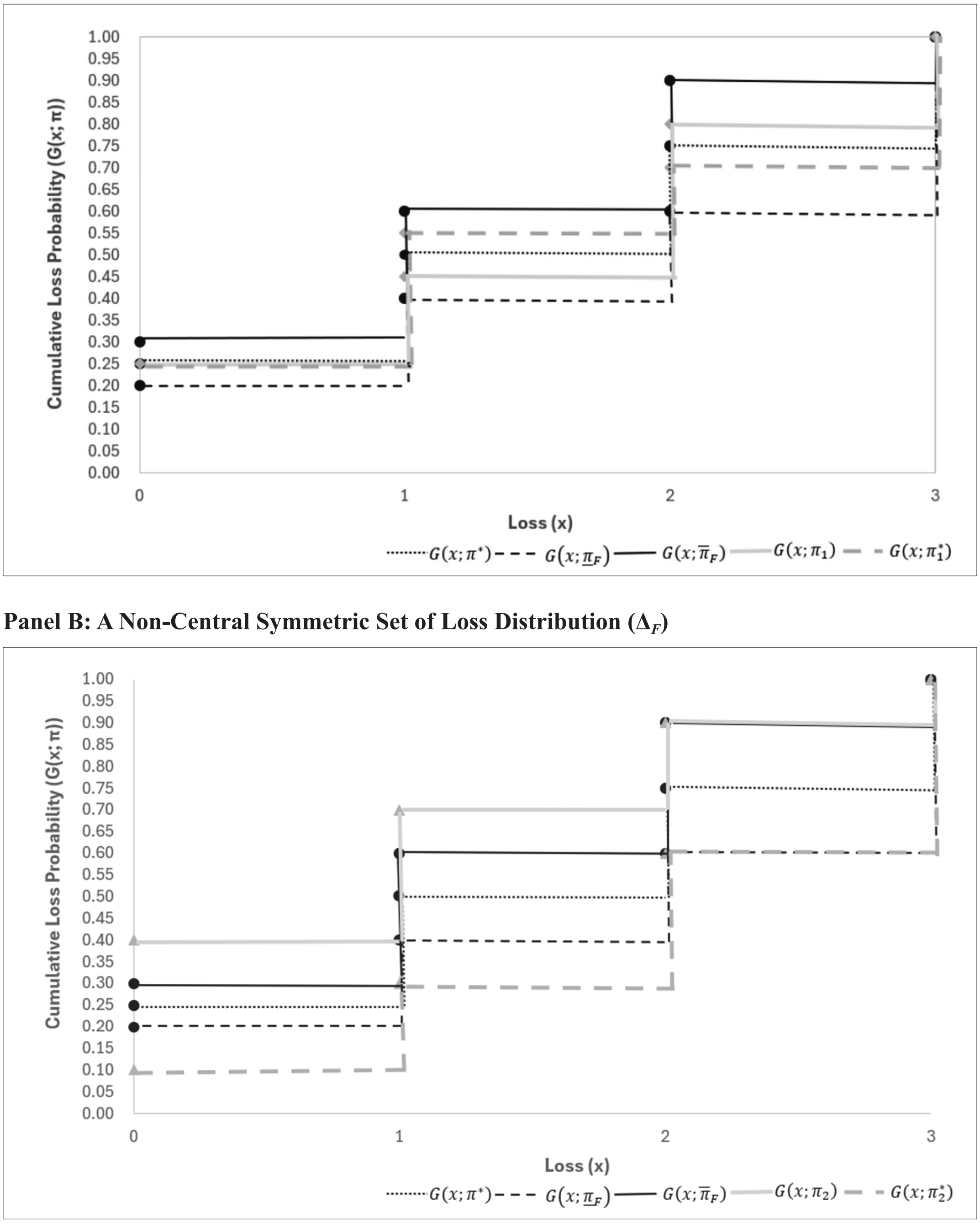

Figure 1 Examples of Central and Non-Central Symmetry of the Loss Distribution Sets

Note: In Panels A and B of Figure 1, the horizontal axis represents a discrete loss x taking a value in the set of {0,1,2,3} and the vertical axis represents the cumulative loss probability at each value of x.

concerning ambiguity-neutral preferences is not influenced by the presence of ambiguity and to characterize ambiguity neutrality with a specific value of α (Jewitt and Mukerji, 2017; Dietz and Walker, 2019; Huang, 2025). Consistent with Assumption 1, when $\alpha = \frac{1}{2}$, the loss distribution is $G(x;\pi^*)$ under ambiguity neutrality. Under Assumption 3, $G(x;\underline{\pi}_F)$ and $G(x;\overline{\pi}_F)$ are the worst and best loss distributions, respectively. We make this assumption to identify the minimal and maximal expected utility in decision-making.[13] Finally, Assumptions 2 and 3 together imply that, for an increasing utility function, $G(x;\underline{\pi}_F) \leq G(x;\pi^*) \leq G(x;\overline{\pi}_F)$ for all $x \in [0,L]$.

Under Assumptions 1–3, the individual decides the optimal $I(x)$ and P to maximize his/her utility in the presence of ambiguity, which is formulated by the α-maxmin model as

$$\max_{I(x)\geq 0,P} \left(\alpha \min_{\pi\in\Pi_F} v(I(x);u,z,\alpha,G,\pi) + (1-\alpha)\max_{\pi\in\Pi_F} v(I(x);u,z,\alpha,G,\pi)\right), \tag{2}$$

subject to P as defined in Equation (1). The term to be maximized in the objective function (2) can be rearranged as

$$v(I(x);u,z,\alpha,G,\underline{\pi}_F,\overline{\pi}_F) = \int_0^L u\,(w-x-P+I(x))\, d\left[\alpha G(x;\underline{\pi}_F) + (1-\alpha)G(x;\overline{\pi}_F)\right], \tag{3}$$

where the utility function u, is assumed to be continuous and differentiable, with $u' > 0$ and $u'' < 0$ representing risk aversion. Moreover, $\alpha \in (\frac{1}{2},1]$ represents ambiguity aversion, and $\alpha G(x;\underline{\pi}_F) + (1 - \alpha)G(x;\overline{\pi}_F)$ can be viewed as the loss distribution distorted by ambiguity aversion under ambiguity.

2.2 The Optimal Insurance Contract

In the absence of ambiguity, under Assumption 1, the loss distribution is $G(x;\pi^*)$.

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13 Under the smooth ambiguity aversion model, Alary et al. (2013) imply a similar relationship between the expected utility and the decision maker's beliefs, with a negative sign arising from the assumption that the loss probability increases with the belief. Gollier (2014) implies a similar relationship with a positive sign by assuming the loss distribution decreased with the belief. If we assume that $\frac{\partial v(I(x);u,z,\alpha,G,\pi)}{\partial \pi} \leq 0$ like Alary et al. (2013), then the worst and best loss distributions are $G(x;\overline{\pi}_F)$ and $G(x;\underline{\pi}_F)$, respectively. On the other hand, under the maxmin expected utility, Birghila et al. (2023) find a closed-form solution for the probability in the worst case and obtain the probability through a numerical approach.

Arrow (1971) proves that, for a risk-averse individual who maximizes his/her expected utility, the optimal insurance contract is a straight deductible $D^* \in (0,L)$ such that $I(x) = \max(0, x - D^*)$ for all $x \in [0,L]$. Under the deductible design, for a loss $x \leq D^*$, the individual obtains no indemnity and takes on all the loss. For a loss $x > D^*$, the individual obtains the partial indemnity $x - D^*$ and suffers a loss limited to D^*. In the absence of ambiguity, this result holds for risk-averse individuals with different ambiguity preferences.

In the presence of ambiguity, we examine the optimality of a straight deductible. Since a risk-averse and ambiguity-neutral individual is insensitive to ambiguity, the straight deductible D^* is still optimal in the presence of ambiguity. The result is described in the following proposition:

Proposition 1: Suppose that ambiguity preferences can be described by an α-maxmin model. Under Assumptions 1–3, for a risk-averse and ambiguity-neutral individual, the optimally straight deductible $D^* \in [0,L]$ in the absence of ambiguity such that $I(x) = \max(0, x - D^*)$ for all $x \in [0,L]$ remains optimal after the introduction of ambiguity for all $x \in [0,L]$.

Proof: The proof is explained as follows.

This result can be obtained directly from the objective function (3) with $\alpha = \frac{1}{2}$; thus, the loss distribution is $G(x;\pi^*)$ under Assumption 2. Following the standard approach to solve the optimal insurance contract (e.g., Arrow, 1971; Gollier, 2014), the optimum is characterized by three first-order conditions (FOCs):

$$u'(w - x - P(D^*) + I(x))\, dG(x;\pi^*) \leq \lambda(1+\tau) dG(x;\pi^*), \text{for all } x \in [0,L], \quad (4)$$

with an equality when $I(x) > 0$,

$$\int_0^L u'\,(w - x - P(D^*) + I(x))\, dG(x;\pi^*) = \lambda, \quad (5)$$

$$(1+\tau)\int_{D^*}^L (x - D^*)\, dG(x;\pi^*) = P(D^*), \quad (6)$$

where λ is the Lagrange multiplier of the premium constraint (1).

For a risk- and ambiguity-averse individual in the presence of ambiguity, we first show that, under the optimal insurance contract, not all the losses will be reimbursed by the insurer when they occur. We present the result in the following proposition:

Proposition 2: Suppose that ambiguity preferences can be described by an α-maxmin model. Under Assumptions 1–3, for a risk- and ambiguity-averse individual, there exists at least one $x \in [0,L]$ such that $I(x) = 0$ under the optimal insurance contract after the introduction of ambiguity for all $x \in [0,L]$.

Proof: See Appendix A.

The results of Propositions 1 and 2 are also shown by Gollier (2014) under the smooth ambiguity aversion model.

Next, we consider when a straight deductible is optimal for the risk- and ambiguity-averse individual after the introduction of ambiguity for all $x \in [0,L]$.

Proposition 3: Suppose that ambiguity preferences can be described by an α-maxmin model. Under Assumptions 1–3, for a risk- and ambiguity-averse individual, there exists a straight deductible $D_F^* \in (0,L)$ such that $I(x) = \max(0,x - D_F^*)$ for all $x \in [0,L]$ which is optimal after the introduction of ambiguity for all $x \in [0,L]$ if the degree of ambiguity is sufficiently small.

Proof: See Appendix B.

By the continuity of the insurance contract, we prove that a small deviation from π^* is sufficient to achieve the optimal straight deductible for the risk- and ambiguity-averse individual. Following Huang (2025), the size of the set Π_F measures the degree of ambiguity according to the α-maxmin model.[14] Under Assumptions 1 and 2, when π deviates more from π^*, the individual perceives more ambiguity. As long as the degree of ambiguity is sufficiently small, the optimal insurance contract is a straight deductible. Gollier (2014) also shows the optimality of a straight deductible, albeit under the

14 This ambiguity measure is explained detailedly in Section 3.

sufficient condition of a small degree of ambiguity aversion. Specifically, when the ambiguity exists for losses below D^*, the smooth ambiguity aversion model implies that a straight deductible remains optimal under a small degree of ambiguity aversion. Our result is consistent with the observational equivalence between the effects of changes in the individual's beliefs and ambiguity aversion on optimal insurance coverage noted by Gollier (2011, 2014).

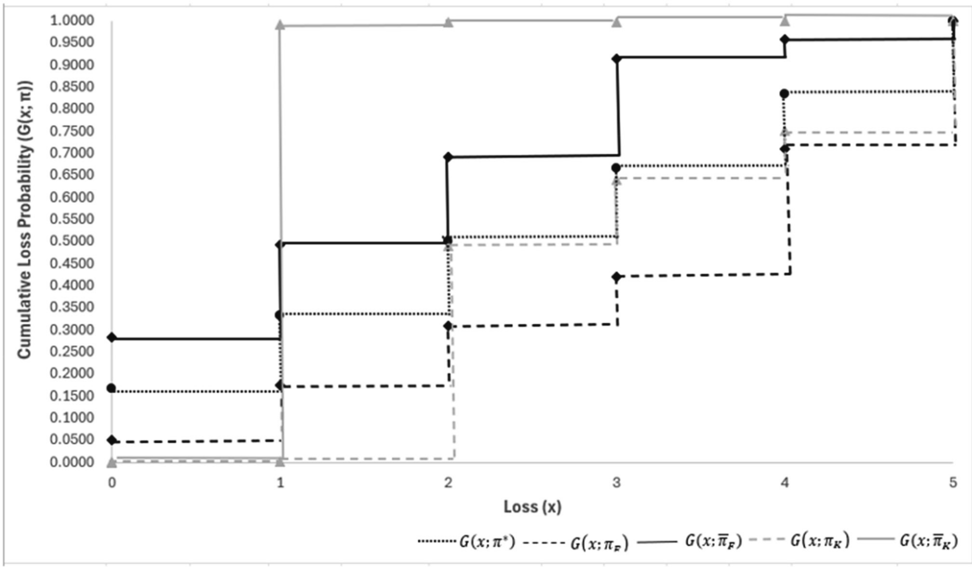
We illustrate Proposition 3 with a numerical example. In Figure 2, Panel A presents all the cumulative loss distribution functions, and Panel B presents the optimal indemnity in the presence of ambiguity. Consider a discrete loss x that takes a value in the set {0,1,2,3,4,5} and an initial wealth $w = 6$. The individual's risk aversion and ambiguity aversion are measured by the constant absolute risk aversion function (CARA) $u(z) = \frac{\sqrt{z}}{2}$ with the CARA coefficient $\gamma = 0.5$, as assumed by Gollier (2014) (Proposition 5), and $\alpha = 0.56$, as estimated by Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2015) via survey. The premium loading factor is $\tau = 0.2$. In the absence of ambiguity, the loss distribution (the dark dotted line in Panel A) is $G(x;\pi^*)$, with the set of cumulative loss probabilities {0.1667,0.3334,0.5001,0.6668,0.8335,1}. As expected, the optimal insurance contract is a straight deductible, with a deductible level of $D^* = 3.42$ (the dark dotted line in Panel B).

In the presence of ambiguity, the loss distributions considered are $G(x;\underline{\pi}_F)$ and $G(x;\overline{\pi}_F)$, represented in Panel A by the dark dashed and solid lines, respectively. For $G(x;\underline{\pi}_F)$, the set of cumulative loss probabilities is {0.0500,0.1734,0.3094,0.4194,0.7094,1}. For $G(x;\overline{\pi}_F)$, the set of cumulative loss probabilities is {0.2834,0.4934,0.6908, 0.9142,0.9576,1}. Under these loss distributions, Assumptions 1–3 hold. As predicted by Proposition 3, the optimal insurance contract is also a straight deductible, albeit with a lower deductible level of $D_F^* = 3.11$ in this example (the dark solid line in Panel B).

As a counterexample, the light dashed and solid lines in Panel A show the loss distributions $G(x;\underline{\pi}_K)$ and $G(x;\overline{\pi}_K)$, respectively. Their sets of cumulative loss probabilities are {0,0.0001,0.4901,0.6401,0.7506,1} and {0.0001,0.9914,0.9964,0.9984,0.9996,1}, respectively. The size of the set, Δ_K, with boundaries $G(x;\underline{\pi}_K)$ and $G(x;\overline{\pi}_K)$, is much larger than Δ_F. Moreover, Assumption 2 does not hold in this case. The optimal contract (the light dashed line in Panel B) is not a straight deductible.

The following comparative statics of an ambiguity increase are based on Proposition

Panel A: The Cumulative Loss Distribution Functions ($G(x;\pi^*)$, $G(x;\underline{\pi}_F)$, $G(x;\overline{\pi}_F)$, $G(x;\underline{\pi}_K)$, $G(x;\overline{\pi}_K)$)



Panel B: The Optimal Indemnity in the Presence of Ambiguity

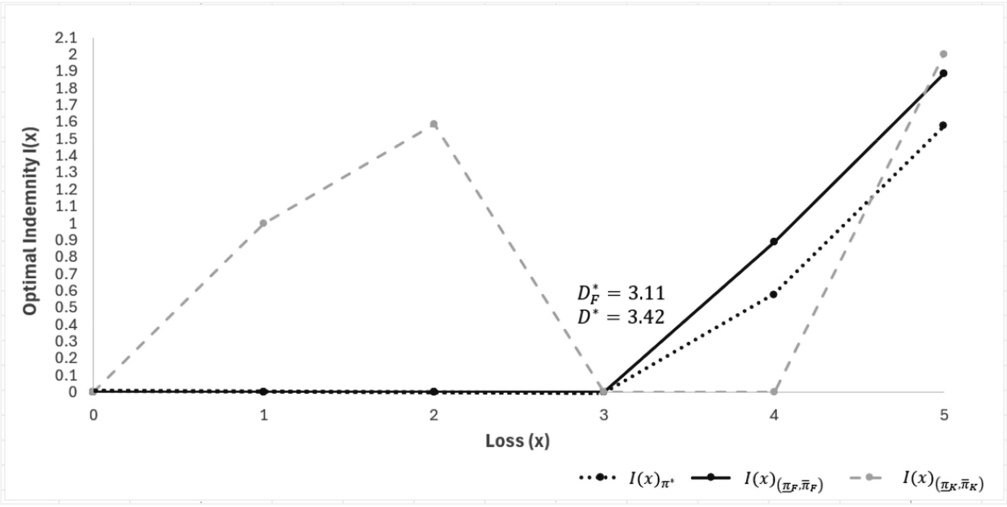


Figure 2 Examples of the Optimal Indemnity in the Presence of Ambiguity under Risk- and Ambiguity-Aversion

Note: In Panel A of Figure 2, the horizontal axis represents a discrete loss *x* taking a value in the set {0,1,2,3,4,5} and the vertical axis represents the cumulative loss probability at each value of *x*. In Panel B of Figure 2, the horizontal axis represents the discrete loss *x* and the vertical axis represents the optimal indemnity under the loss distributions in Panel A.

3, that is, on the assumption that a straight deductible is optimal. Thus, the problem for the individual is choosing a deductible level D_F that maximizes $v(I(x);u,z,\alpha,G,\underline{\pi}_F,\overline{\pi}_F)$ as follows:

$$\begin{aligned}\max_{DF}\ & v(I(x);\ u,z,\alpha,G,\underline{\pi}_F,\overline{\pi}_F)\\ &= \int_0^{D_F} u(w-x-P(D_F))\, d\big[\alpha G(x;\underline{\pi}_F) + (1-\alpha)G(x;\overline{\pi}_F)\big]\\ &\quad + u(w-D_F-P(D_F))\Big[1-\Big(\alpha G\big(D_F;\underline{\pi}_F\big)+(1-\alpha)G(D_F;\overline{\pi}_F)\Big)\Big],\end{aligned} \tag{7}$$

where $P(D_F)=(1+\tau)\int_{D^*}^{L}(x-D^*)\,dG(x;\pi^*)$. Denote D_F^* as the optimal deductible level when $\pi\in\Pi_F$. [15] The FOC is

$$\begin{aligned}&(1+\tau)\big(1-G(D_F^*;\pi^*)\big)\int_0^{D_F^*} u'(w-x-P\,(D_F^*))\,d\big[\alpha G(x;\underline{\pi}_F)+(1-\alpha)G(x;\overline{\pi}_F)\big]\\ &\quad -[1-(1+\tau)(1-\ G(D_F^*;\pi^*))]\,u'(w-D_F^*-P(D_F^*))\\ &\quad \times\big[1-(\alpha\ G\big(D_F^*;\underline{\pi}_F\big)+(1-\alpha)G(D_F^*;\pi^*))\big]=0.\end{aligned} \tag{8}$$

Note that, to drive FOC (8) to zero at D_F^*, $1-(1+\tau)(1-G(D_F^*;\pi^*))$ must be positive. Based on the left-hand side of FOC (8), lowering the deductible level under the distorted loss distribution could affect two aspects. The first term on the left-hand side is the expected marginal utility cost in the uncovered loss state due to a higher premium. The second term on the left-hand side is the net marginal utility benefit in the loss state covered by the partial indemnity due to a higher premium and higher insurance coverage.

3. An Ambiguity Increase

We now define an ambiguity increase. Ghirardato et al. (2004) propose the α-maxmin model and show that the size of the set of probabilities of acts represents the degree of ambiguity perceived by a decision maker. The larger the set, the greater the ambiguity perceived. Using the maxmin model, Koufopoulos and Kozhan (2014, 2016) define

15 Because $u'>0$ and $\tau>0$, D_F^* is internal. The second-order condition holds if at the initial deductible level, the individual's loss probability density function distorted by ambiguity aversion relative to the insurer's loss probability density function is small enough that $\frac{d[\alpha G(x;\underline{\pi}_F)+(1-\alpha)G(x;\overline{\pi}_F)]}{dG(D_F;\pi^*)}<1+\tau$.

the interval of accident probabilities as the degree of ambiguity to model asymmetric information and the uncertainty of the accident probabilities in the equilibrium of insurance markets. The longer the interval, the greater the ambiguity perceived. Based on these approaches, the size of Π_F reflects the degree of ambiguity. Accordingly, as the set Π_F becomes larger, the individual experiences an ambiguity increase.[16]

Next, we define another set of π: $\Pi_T = \{\pi | \underline{\pi}_T \leq \pi \leq \overline{\pi}_T\} \supseteq \Pi_F$. Accordingly, $\pi^* \in \Pi_F$ implies that $\pi^* \in \Pi_T$. We say the individual faces an ambiguity increase when his/her belief shifts from Π_F to Π_T. Additionally, let Δ_T denote the set $G(x;\pi)$ when $\pi \in \Pi_T$. We also let Assumptions 1–3 hold for Δ_T. Thus, Δ_T shares the same center $G(x;\pi^*)$ as Δ_F. Formally, we define an ambiguity increase as follows.

Definition 2: Under Assumptions 2 and 3, an individual experiences an ambiguity increase if his/her belief shifts from Π_F to Π_T, where $\pi^* \in \Pi_F \subseteq \Pi_T$, and Δ_F and Δ_T share the same center $G(x;\pi^*)$.

Due to ambiguity neutrality, the loss distribution $G(x;\pi^*)$ used by the insurer for premium pricing is not affected by an ambiguity increase as defined above. Thus, a change in the premium results solely from a change in the deductible level chosen by the individual.

To provide a better understanding of Definition 2, Figure 3 shows a numerical example similar to the central symmetry example in Figure 1.[17] Here, $G(x;\pi^*)$ (the dark dotted line), $G(x;\underline{\pi}_F)$ (the dark dashed line), and $G(x;\overline{\pi}_F)$ (the dark solid line) are unchanged. Suppose that the individual's belief changes from Π_F to Π_T for an exogenous reason. The sets of cumulative loss probabilities for $G(x;\underline{\pi}_T)$ (the light dashed line) and $G(x;\overline{\pi}_T)$ (the light solid line) are $\{0.1,0.3,0.6,1\}$ and $\{0.4,0.7,0.9,1\}$, respectively. In this case, Assumptions 2 and 3 still hold, and figure shows that $\Delta_T \supset \Delta_F$ with the same center $G(x;\pi^*)$. Therefore, the belief change is an ambiguity increase as described in Definition 2.

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16 Huang and Tzeng (2018) use a different definition of an ambiguity increase under the α-maxmin model. They assume that there are only two possible loss distributions, and the probability π of the better one (in terms of FSD) follows a distribution. The ambiguity increase is defined on the distribution of π as the Nth-degree risk increase of Ekern (1980) preserving the α-weighted average of π. Actually, their definition implies a broader set of π than ours.

17 We appreciate this suggestion proposed by an anonymous reviewer.

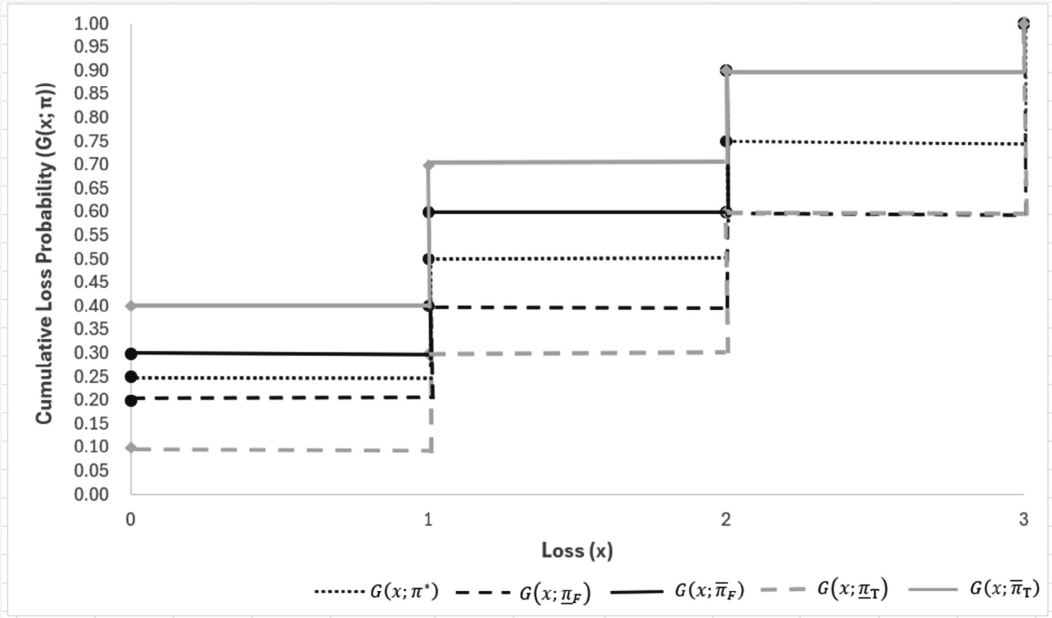

Figure 3 An Example of an Ambiguity Increase as Described in Definition 2

Note: In Figure 3, the horizontal axis represents a discrete loss *x* taking a value in the set of {0,1,2,3} and the vertical axis represents the cumulative loss probability at each value of *x*.

3.1 A Specific Ambiguity Increase

In this section, we discuss a specific ambiguity increase that preserves the cumulative loss probability at D^*_F. Powers and Tzeng (2001) make a similar assumption on the risk increase in the absence of ambiguity and study its impact on the optimal deductible. In their terminology, we assume that, at D^*_F, $G(x;\underline{\pi}_F)$ and $G(x;\underline{\pi}_T)$ are stochastically equivalent, meaning that $G(x;\overline{\pi}_F)$ and $G(x;\overline{\pi}_T)$ are stochastically equivalent in this paper.[18] We define the specific ambiguity increase as follows.

Definition 3: Under Assumptions 2 and 3, an individual experiences a specific ambiguity

18 Powers and Tzeng (2001) note that a risk change can affect the insurer's pricing and the optimal deductible chosen by the insured. They consider a mean-preserving risk increase for the loss above D^* to ensure that any change in the optimal deductible level purely results from a risk change for the loss below D^* instead of a price change. Since an ambiguity change does not affect the insurer's pricing for the risk in our settings, we do not make this assumption.

increase if the ambiguity increase is consistent with Definition 2 and preserves the cumulative loss probability at the initial optimal deductible D^*_F.

Under this assumption, at D^*_F, the expected utility in the loss state covered by the insurer is unaffected by the ambiguity increase. In other words, the net marginal utility benefit of lowering the deductible is unchanged. This leads to the same result as the assumption of an ambiguity increase affecting the loss below D^*_F; that is, $G(x;\underline{\pi}_F) = G(x;\underline{\pi}_T)$ and $G(x;\overline{\pi}_F) = G(x;\overline{\pi}_T)$ for losses above D^*_F, which implies a specific ambiguity increase. Similarly, Gollier (2014) considers the ambiguity affecting losses below D^* to study the optimal insurance contract under ambiguity aversion formulated by the smooth ambiguity aversion model.

To clarify Definition 3, we present a numerical example in Figure 4.[19] Consider a discrete loss x that takes a value in the set $\{0,1,2,3,4\}$ and $w = 5$. The full specifications of the loss distributions are provided in Panels A and B in Appendix C. We assume $\tau = 0.2$, the CARA coefficient $\gamma = 0.5$, and $\alpha = 0.56$, as assumed for the numerical example of Proposition 3. Before an ambiguity increase, under the loss distributions $G(x;\underline{\pi}_F)$ and $G(x;\overline{\pi}_F)$ (the dark dashed and solid lines, respectively) with center $G(x;\pi^*)$ (the dark dotted line), we have $D^*_F = 2$. After an ambiguity increase, the loss distributions become $G(x;\underline{\pi}_T)$ and $G(x;\overline{\pi}_T)$ (the light dashed and solid lines, respectively) with the same center. In this case, Assumptions 2 and 3 hold. The figure indicates that $G(2;\underline{\pi}_F) = G(2;\underline{\pi}_T) = 0.9285$ and $G(2;\overline{\pi}_F) = G(2;\overline{\pi}_T) = 0.9315$, which satisfies Definition 3. Thus, the ambiguity increase is the specific one.

Suppose that a specific ambiguity increase occurs. We show how a risk- and ambiguity-averse individual respond to this specific ambiguity increase in the following proposition, where D^*_T denotes the optimal deductible after the ambiguity increase.

Proposition 4: Suppose that ambiguity preferences can be described by an α-maxmin model. Under Assumptions 1–3, a risk- and ambiguity-averse individual reacts to a specific ambiguity increase as defined in Definition 3 as follows:

19 We are grateful for an anonymous reviewer's suggestion to illustrate this definition with a numerical example.

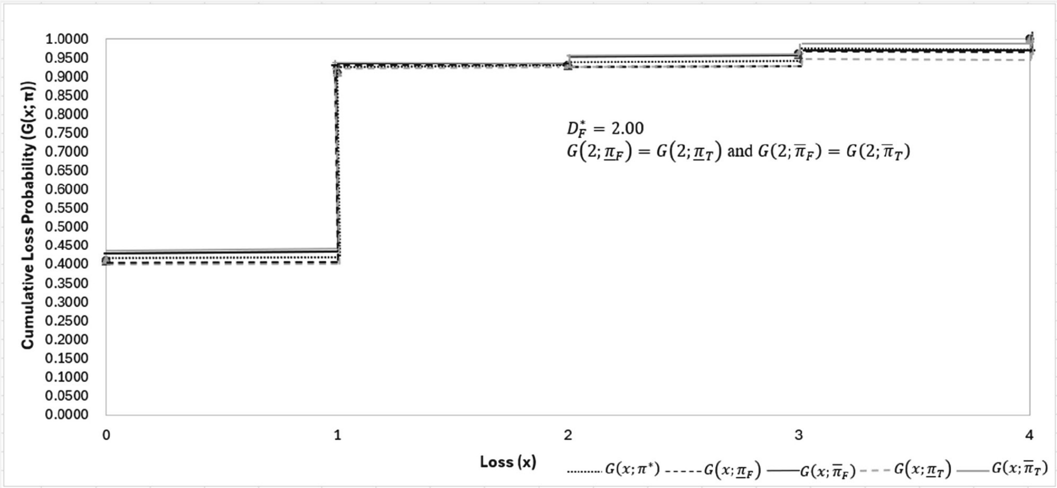

Figure 4 An Example of a Specific Ambiguity Increase as Described in Definition 3

Note: In Figure 4, the horizontal axis represents a discrete loss *x* taking a value in the set of {0,1,2,3,4} and the vertical axis represents the cumulative loss probability at each value of *x*.

(1) $D_T^* = D_F^*$ if and only if, $\forall x \in [0, D_F^*]$, $G(x;\underline{\pi}_T) = G(x;\underline{\pi}_F)$ and $G(x;\overline{\pi}_T) = G(x;\overline{\pi}_F)$.

(2) When Assumption 2 is relaxed so that the center of Δ_T is not $G(x;\pi^*)$, $D_T^* \leq D_F^*$ if and only if, $\forall x \in [0, D_F^*]$, $G(x;\underline{\pi}_T) = G(x;\underline{\pi}_F)$ and $G(x;\overline{\pi}_T) - G(x;\overline{\pi}_F) \geq 0$ with strict inequality for some x.

Proof: See Appendix D.

Case 1 of Proposition 4 states that, in the face of a specific ambiguity increase, a risk- and ambiguity-averse individual with the α-maxmin preference still chooses D_F^* as the optimal deductible level when the specific ambiguity increase does not affect losses below D_F^*. The intuition is that, under the deductible D_F^*, any losses above D_F^* are covered by the insurer and are therefore treated as an unambiguous risk. In other words, only the loss below D_F^* is an ambiguous risk for the individual. Since the specific ambiguity increase has no impact on losses below D_F^*, the individual chooses the same deductible level to continue facing an unambiguous risk. This result is consistent with the rationale behind the results reported by Gollier (2014) under the smooth ambiguity aversion model. Gollier (2014) finds that the optimal insurance contract under ambiguity aversion is the same as under ambiguity neutrality when the ambiguity only affects loses above D^*.

Next, we investigate when a risk- and ambiguity-averse individual facing a specific ambiguity increase lowers the optimal deductible (i.e., increases insurance coverage), as this matters to both the individual and the insurer. We obtain the determining condition stated in Case 2 of Proposition 4 when relaxing Assumption 2 by allowing for Δ_T with a different center from Δ_F while keeping $G(x;\pi^*) \in \Delta_F \subset \Delta_T$. We refer to this as a *nonspecific* ambiguity increase. Under this type of ambiguity increase, the individual chooses a lower optimal deductible than D^*_F when the worst loss distribution remains unaffected while the best loss distribution (and the loss distribution distorted by α) deteriorates in the sense of FSD for losses below D^*_F. The intuition is as follows. An extremely ambiguity-averse individual ($\alpha = 1$) who pessimistically believes the worst loss distribution will be realized still chooses D^*_F since the uncovered loss $x \in [0,D^*_F]$ is unaffected by the nonspecific ambiguity increase. On the other hand, a non-extremely ambiguity-averse individual ($\alpha \in \left(\frac{1}{2},1\right)$) believes that the realized loss distribution could be the worst or best one. The worst distribution remains unchanged, whereas under the best distribution, the mean loss is reduced. This means that at D^*_F, the individual is now more likely to take the full loss, without any indemnity from the insurer. Based on FOC (8), after the nonspecific ambiguity increase, lowering the deductible level at D^*_F reduces the expected marginal utility cost in the uncovered loss state while leaving the net marginal utility benefit covered by the partial indemnity unchanged. Therefore, it is optimal for the individual to lower the deductible.

We demonstrate Proposition 4 in Figure 5 through a numerical example of a specific ambiguity increase. The full settings of the example are reported in Appendix C, Table 1. To show the key results clearly, we zoom in on the point where the individual starts to receive positive indemnity, indicating the optimal deductible level. In this example, in the absence of ambiguity, $D^* = 2.01$ (the dark dotted line) and introducing ambiguity lowers the optimal deductible to $D^*_F = 2$ (the dark solid line). The loss distributions after the specific ambiguity increase, $G(x;\underline{\pi}_T)$ and $G(x;\overline{\pi}_T)$ satisfy Assumptions 2 and 3 and Case 1 of Proposition 4 (Panel C of Table 1). Thus, after the specific ambiguity increase, the optimal deductible level does not change ($D^*_T = 2$, the dark solid line).

Let us consider another example. Here, a nonspecific ambiguity increase occurs, and

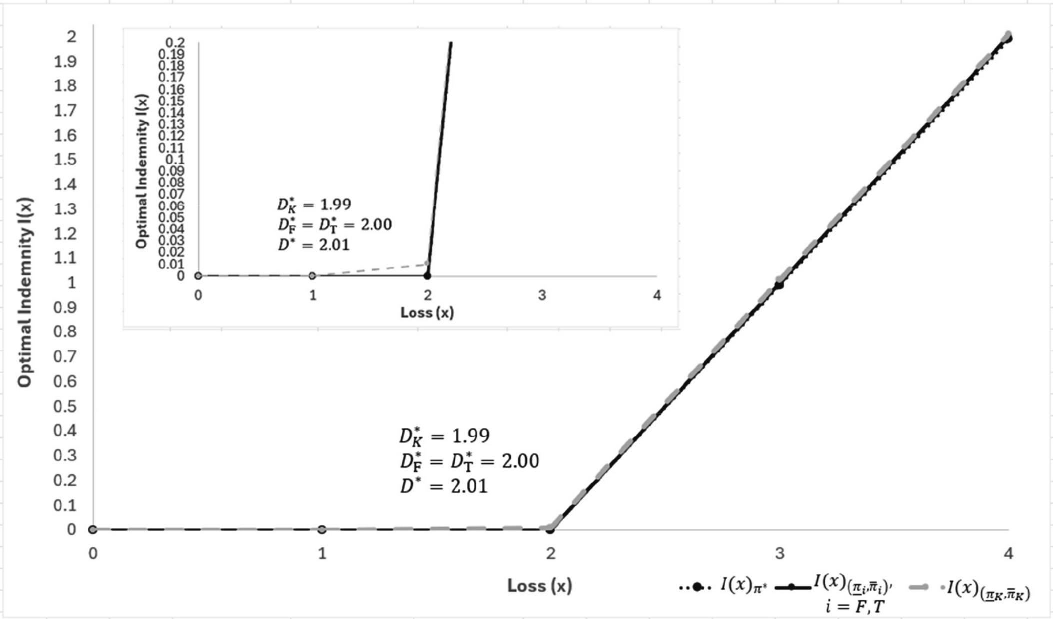

Figure 5 An Example of Proposition 4

Note: In Figure 5, the horizontal axis represents a discrete loss *x* taking a value in the set of {0,1,2,3,4} and the vertical axis represents the optimal indemnity at each value of *x*.

the associated set of loss distributions (Δ_K) does not have the same center as before (Panel C of Table 1), that is, Assumption 2 is violated. In addition, $G(x;\underline{\pi}_K)$ and $G(x;\overline{\pi}_K)$ satisfy Case 2 of Proposition 4. From Figure 5, we can see that the nonspecific ambiguity increase decreases the optimal deductible level to $D_K^* = 1.99$ (the light dashed line).

Our determining conditions are comparable to those reported in previous studies of how risk changes affect a risk-averse individual's optimal deductible level when the loss distribution is certain. Eeckhoudt, Gollier, and Schlesinger (1991) show that a mean-preserving risk change involving FSD deterioration leads to a lower optimal deductible level if the individual exhibits decreasing or constant absolute risk aversion. Powers and Tzeng (2001) find the determining conditions for a risk change to decrease the optimal deductible level under risk aversion. In particular, their determining conditions involve FSD deterioration of the loss distribution below D^* while preserving the cumulative loss probability at D^*. Extending this to the case of uncertain loss distributions, we find analogous determining conditions for a risk- and ambiguity-averse individual under the similar assumption of the preserving cumulative loss probability made for the specific ambiguity increase.

Our results augment the literature on ambiguity as follows. First, our results extend the work of Birghila et al. (2023). They show that under the maxmin model, the optimal insurance contracts with and without the sabotage condition imposed on the indemnity function are in the deductible form, although with different shapes of the optimal indemnity function. We study how ambiguity affects the optimal deductible level under the more general α-maxmin model, without assuming the sabotage condition. In terms of studying the optimal deductible level when the risk is ambiguous, our results can be treated as a dual version of Alary et al. (2013) and Gollier (2014) but under a different model. Given certain ambiguity structures, these authors study how the optimal deductible changes under ambiguity aversion compared to ambiguity neutrality for individuals with smooth-ambiguity-aversion preferences. Alary et al. (2013) show that when ambiguity occurs only in the no-loss state, ambiguity aversion reduces the optimal deductible relative to ambiguity neutrality under the smooth ambiguity aversion model. Using the same model, Gollier (2014) studies a different ambiguity structure. When ambiguity affects only losses below D^*, he shows that, for possible loss distributions ranked according to FSD, introducing ambiguity aversion is sufficient to raise the optimal deductible. On the other hand, given ambiguity aversion, we examine how the optimal deductible changes under an ambiguity increase for α-maxmin preferences.

3.2 A General Ambiguity Increase

We now discuss a general ambiguity increase. In this case, we do not preserve the cumulative loss probability at D^*_F, which is required in the case of a specific ambiguity increase. The general ambiguity increase is described in Definition 2. Let $\dot{G}_i\,(x,\alpha)$ denote $\alpha G(x;\underline{\pi}_i) + (1 - \alpha)G(x;\overline{\pi}_i)$, where $i = F$ or T. The exogenous variables are suppressed to simplify notations. Accordingly, FOC (8) divided by $(1 + \tau)(1 - G(D^*_F;\pi^*))$ becomes

$$\int_0^{D_F^*} u'\left(w - x - P(D_F^*\,)\right) d\dot{G}_F(x,\alpha) - \left[\frac{1}{(1+\tau)\left(1-G\left(D_F^*;\pi^*\right)\right)} - 1\right] u'\left(w - D_F^* - P(D_F^*)\right)\left[1 - \dot{G}_F(D_F^*,\alpha)\right] = 0. \tag{9}$$

As shown in Appendix E, finding the determining condition under which the general ambiguity increase lowers the optimal deductible level is equivalent to finding when $Q(x,\alpha) \le 0$ for all $x \in [0,D^*_F]$ and $\alpha \in (\frac{1}{2},1]$, where $Q(x,\alpha)$ is defined as follows:

$$Q(x,\alpha) = \left[\dot{G}_T(D^*_F;\alpha) - \dot{G}_T(x;\alpha)\right]\left[1 - \dot{G}_F(D^*_F;\alpha)\right] - \left[\dot{G}_F(D^*_F;\alpha) - \dot{G}_F(x;\alpha)\right]\left[1 - \dot{G}_T(D^*_F;\alpha)\right]. \tag{10}$$

Note that $Q(x,\alpha)$ is a quadratic function of $\alpha \in [0,1]$ for any $x \in [0,D^*_F]$. Based on this property, we formulate the following proposition.

Proposition 5: Suppose that ambiguity preferences can be described by an α-maxmin model. Under Assumptions 1–3, a risk- and ambiguity-averse individual experiencing a general ambiguity increase (as described in Definition 2) lowers the optimal deductible level if and only if, at any $x_q \in [0,D^*_F]$, there exists $\alpha^*(x_q) \in [0,1]$ at which $Q(x_q,\alpha^*(x_q))$ is a local maximum or minimum such that one of the following conditions holds for all $\alpha \in (\frac{1}{2},1]$:

(1) $Q(x_q,\frac{1}{2}+\varepsilon) \le 0$ when $Q(x_q,\alpha^*(x_q))$ is a local maximum at $\alpha^*(x_q) \in [0,\frac{1}{2}]$, where $0 < \varepsilon \le \frac{1}{2}$.

(2) $Q(x_q,1) \le 0$ when $Q(x_q,\alpha^*(x_q))$ is a local minimum at $\alpha^*(x_q) \in [0,\frac{1}{2}]$.

(3) $Q(x_q,\alpha^*(x_q)) \le 0$ when $Q(x_q,\alpha^*(x_q))$ is a local maximum at $\alpha^*(x_q) \in (\frac{1}{2},1]$.

(4) $Q(x_q,\frac{1}{2}+\varepsilon) \le 0$ and $Q(x_q,1) \le 0$ when $Q(x_q,\alpha^*(x_q))$ is a local minimum at $\alpha^*(x_q) \in (\frac{1}{2},1)$, where $0 < \varepsilon \le \frac{1}{2}$.

(5) $Q(x_q,\frac{1}{2}+\varepsilon) \le 0$ when $Q(x_q,\alpha^*(x_q))$ is a local minimum at $\alpha^*(x_q) = 1$, where $0 < \varepsilon \le \frac{1}{2}$.

Proof: See Appendix E.

We obtain the determining conditions for reducing the optimal deductible under a general ambiguity increase for the five cases in Proposition 5. Each condition applies to a different case depending on which interval $\alpha^*(x_q)$ lies in. To provide a better understanding of our results, we present the five cases in Panels A–E of Figure 6.

Proposition 5 states that, in the face of a general ambiguity increase, a risk- and ambiguity-averse individual with the α-maxmin preference chooses a lower optimal deductible level than D^*_F. This occurs under the distorted loss distribution when, at D^*_F, the odds of receiving partial indemnity increase compared to facing

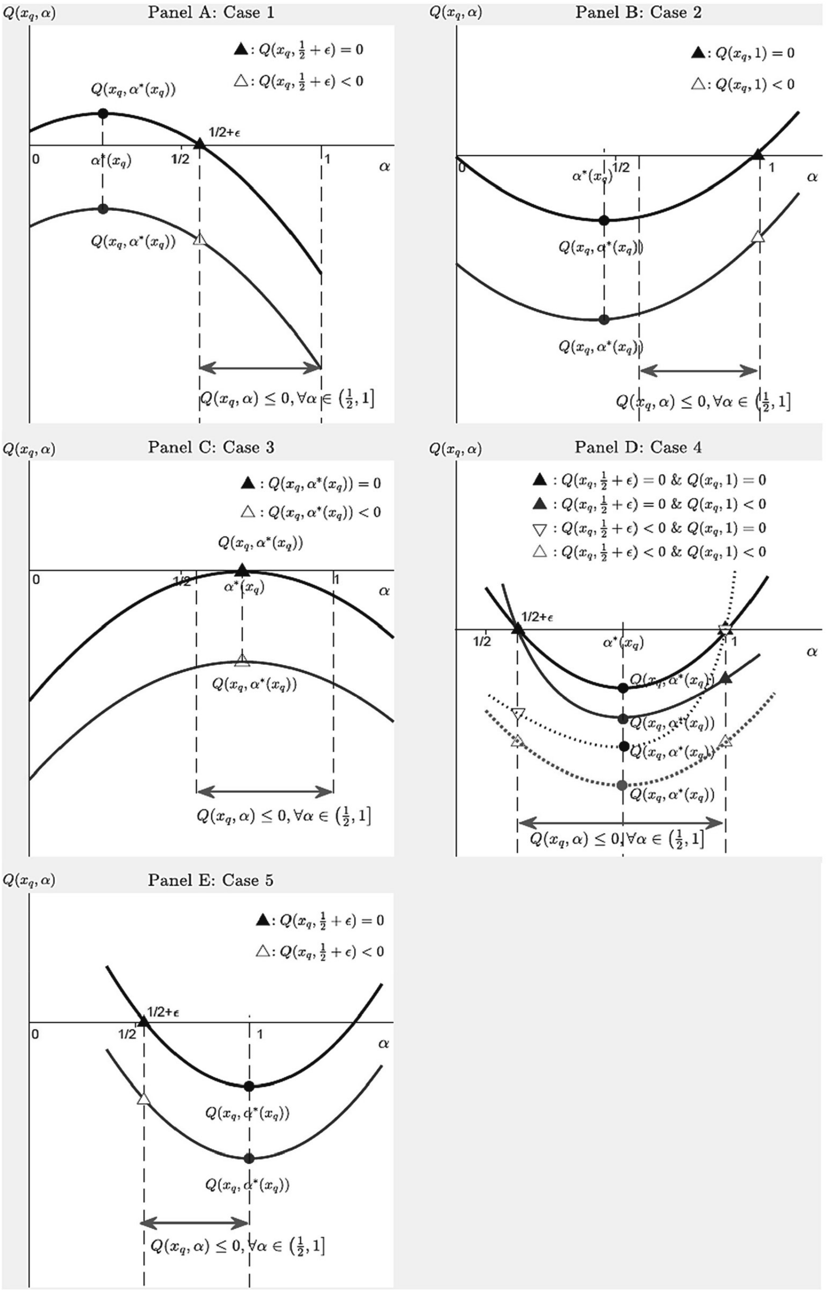

Figure 6 Determining Conditions for a General Ambiguity Increase to Decrease the Optimal Deductible Level

Note: In Figure 6, the horizontal axis represents an α with a value between 0 and 1characterizing ambiguity preference of an individual and the vertical axis represents a quadratic function $Q(x_q,\alpha)$ of α at any loss $x_q \in [0,D_F^*]$.

no indemnity and suffering a large loss between x_q and D_F^*, where the realized loss is $x_q \in [0,D_F^*]$. For example, Condition 2 of Proposition 5, $Q(x_q,1) \leq 0$, is written as $\frac{1-G(D_F^*;\underline{\pi}_T)}{G(D_F^*;\underline{\pi}_T)-G(x_q;\underline{\pi}_T)} \geq \frac{1-G(D_F^*;\underline{\pi}_F)}{G(D_F^*;\underline{\pi}_F)-G(x_q;\underline{\pi}_F)}$ for any $x_q \in [0,D_F^*]$. This means that for an extremely ambiguity-averse individual with $\alpha = 1$, at D_F^*, the odds of obtaining partial indemnity under the worst loss distribution increase after the general ambiguity increase. Condition 5, $Q(x_q, \frac{1}{2}+\varepsilon) \leq 0$, is interpreted similarly, but the odds under the $(\frac{1}{2}+\varepsilon)$ -weighted average of the worst and best loss distributions, where $0 < \varepsilon \leq \frac{1}{2}$, relate to a non-extremely ambiguity-averse individual with $\alpha = \frac{1}{2}+\varepsilon$.

We interpret this result using FOC (9). Accompanying a general ambiguity increase, a risk- and ambiguity-averse individual at D_F^* experiences higher marginal utility in the loss state covered by partial indemnity under the distorted loss distribution. In particular, lowering the deductible level after the ambiguity increase raises the net marginal utility benefit in the indemnified loss state while leaving the expected marginal utility cost in the uncovered loss state unchanged. Accordingly, after the general ambiguity increase, it is optimal for the individual to lower the deductible level.

4. Conclusion

This paper provides the determining conditions that raise the optimal insurance coverage of deductibles after various ambiguity increases. We first prove the optimality of a straight deductible under the α-maxmin model. Then, the specific, nonspecific, and general ambiguity increases are examined in turn for a risk- and ambiguity-averse individual. We focus on the deductible due to its importance and widespread use in both insurance markets and the insurance literature. Our results are necessary and sufficient. Furthermore, our conditions in the case of the nonspecific ambiguity increase are preference-free, thereby facilitating future applications of our findings.

Future research could empirically examine our conditions, and additional determining conditions could be obtained by considering the insurer's risk and ambiguity preferences (e.g., Birghila et al., 2023). Future studies may also consider the nonperformance ambiguity of the insurance contract, which may impact insurance demand (e.g., Peter and

Ying, 2020; Lambregts, van Bruggen, and Bleichrodt, 2021).[20]

Finally, since insurers' financial strength may influence their decision-making (e.g., Weiss, Cheng, and Lin, 2024),[21] future research might examine the heterogeneity of insurers' financial health and assess its effect on our results.

20 In the context of coinsurance, Peter and Ying (2020) prove that introducing ambiguity—defined as uncertainty about the insurer's nonperformance probability rather than uncertainty about the loss probability—reduces insurance demand. Through a laboratory experiment, Lambregts et al. (2021) find evidence supporting the findings of Peter and Ying (2020) for risk-prudent individuals but not for ambiguity-averse individuals.

21 Weiss et al. (2024) find that using an internal actuary to certify loss reserves results in larger under-reported loss reserves than using an external actuary. This result is more severe for financially weak insurers than for financially strong insurers.

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Appendices

Appendix A: Proof of Proposition 2

Due to ambiguity aversion, the first-order conditions (FOCs) (4) and (5) become:

$$u'(w - x - P(D^*) + I(x))d[\alpha G(x;\ \underline{\pi}_F) + (1-\alpha)G(x;\ \overline{\pi}_F)] \ \leq\ \lambda(1+\tau)\ dG(x;\pi^*)$$

for all $x \in [0, L]$, with an equality when $I(x) > 0$,

(A1)

and

$$\int_0^L u'\ (w - x - P(D^*) + I(x))\ d\big[\alpha G(x;\underline{\pi}_F) + (1-\alpha)G(x;\overline{\pi}_F)\big] = \lambda,$$

(A2)

while Condition (6) is unchanged.

Suppose by contradiction that the insurer provides $I(x) > 0$ for all $x \in [0, L]$. Then, Condition (A1) holds at the equality. This means that $\int_0^L u'\ (w - x - P(D^*) + I(x))\ d\big[\alpha G(x;\underline{\pi}_F) + (1-\alpha)G(x;\overline{\pi}_F)\big] = \lambda(1+\tau)$, which contradicts Condition (A2) since $\tau > 0$. As a result, there exists at least one $x \in [0, L]$ that will not be reimbursed by the insurer when it occurs.

Q.E.D.

Appendix B: Proof of Proposition 3

Let $\underline{\pi}_F = \pi^* - \varepsilon$ and $\overline{\pi}_F = \pi^* + \varepsilon$, where ε is positive and small. Then, the degree of ambiguity is represented by the set $[\pi^* - \varepsilon, \pi^* + \varepsilon]$. When $\varepsilon = 0$, this is the case without ambiguity, and Proposition 1 states that D^* is optimal satisfying Conditions (4) to (6).

In the presence of ambiguity, $\varepsilon > 0$. The optimal insurance contract should satisfy Conditions (A1) and (A2). By the continuity of the optimal insurance contract, when $\varepsilon \to 0$, we know that Conditions (A1) and (A2) hold. Accordingly, there exists a straight deductible D_F^* such that $I(x) = \max(0, x - D_F^*)$ for all $x \in [0, L]$ which is optimal.

Q.E.D.

Appendix C: Settings of the Specific Ambiguity Increase in Definition 3 and Proposition 4

The following table details the settings used to demonstrate the example in Definition 3 and Proposition 4.

Table 1 Settings of the Specific Ambiguity Increase in Definition 3 and Proposition 4

Loss (x)	0	1	2	3	4
Panel A: Distribution used by the insurer					
$G(x;\ \pi^*)$	0.4100	0.9100	0.9300	0.9600	1.0000
Panel B: Distribution used by the individual					
Before the specific ambiguity increase: Δ_F					
$G(x;\underline{\pi}_F)$	0.4070	0.9100	0.9285	0.9600	1.0000
$G(x;\overline{\pi}_F)$	0.4130	0.9100	0.9315	0.9600	1.0000
After the specific ambiguity increase: Δ_T					
$G(x;\underline{\pi}_T)$	0.4070	0.9100	0.9285	0.9595	1.0000
$G(x;\overline{\pi}_T)$	0.4130	0.9100	0.9315	0.9605	1.0000
After the nonspecific ambiguity increase: Δ_K					
$G(x;\underline{\pi}_K)$	0.4070	0.9100	0.9285	0.9595	1.0000
$G(x;\overline{\pi}_K)$	0.4800	0.9100	0.9315	0.9615	1.0000
Panel C: Conditions of Proposition 4					
Before vs. after: Δ_F *vs.* Δ_T					
(1) $G(x;\underline{\pi}_T) - G(x;\underline{\pi}_F)$	0.0000	0.0000	0.0000	-0.0005	0.0000
(2) $G(x;\overline{\pi}_T) - G(x;\ \overline{\pi}_F)$	0.0000	0.0000	0.0000	0.0005	0.0000
Before vs. after: Δ_F *vs.* Δ_K					
(1) $G(x;\underline{\pi}_K) - G(x;\underline{\pi}_F)$	0.0000	0.0000	0.0000	-0.0005	0.0000
(2) $G(x;\overline{\pi}_K) - G(x;\overline{\pi}_F)$	0.0670	0.0000	0.0000	0.0015	0.0000

Appendix D: Proof of Proposition 4

Assume that the second-order condition (SOC) holds. Denote D_T^* as the optimal deductible level when $\pi \in \Pi_T$. By Definition 3, $G(D_F^*; \underline{\pi}_F) = G(D_F^*; \underline{\pi}_T)$ and $G(D_F^*; \overline{\pi}_F) = G(D_F^*; \overline{\pi}_T)$. Thus, integrating the left-hand side of FOC (8) by parts yields that $D_T^* \leq D_F^*$ if and only if

$$(1+\tau)\big(1-G(D_F^*;\pi^*)\big)\big[1+(1+\tau)\big(1-G(D_F^*;\pi^*)\big)\big] \times \left\{\int_0^{D_F^*} u''\left(w-x-P(D_F^*)\right) \times \left[\alpha\big(G(x;\underline{\pi}_T)-G(x;\underline{\pi}_F)\big)+(1-\alpha)\big(G(x;\overline{\pi}_T)-G(x;\overline{\pi}_F)\big)\right]\right\} dx \leq 0. \quad \text{(D1)}$$

We first prove that for all $u' > 0$, $u'' < 0$, and $\alpha \in [0,1]$, Equation (D1) holds for all $x \in [0, D_F^*]$ if and only if

$$\alpha\big[G(x;\underline{\pi}_T)-G(x;\underline{\pi}_F)\big]+(1-\alpha)\big[G(x;\overline{\pi}_T)-G(x;\overline{\pi}_F)\big] \geq 0, \quad \text{(D2)}$$

for all $\alpha \in [0,1]$ and $x \in [0, D_F^*]$, as follows.

The *if* part: Since $(1+\tau)(1-G(D_F^*;\pi^*)) > 0$ and $u'' < 0$, if Condition (D2) holds for all $\alpha \in [0,1]$ and $x \in [0, D_F^*]$, then, for all $u' > 0$, $u'' < 0$, and $\alpha \in [0,1]$, Equation (D1) holds for all $x \in [0, D_F^*]$.

The *only if* part: Suppose, by contradiction, that there exists $x_D \in [0, D_F^*]$ such that $\alpha\big[G(x_D;\underline{\pi}_T) - G(x_D;\underline{\pi}_F)\big] + (1-\alpha)[G(x_D;\overline{\pi}_T) - G(x_D;\overline{\pi}_F)] < 0$ for all $\alpha \in [0,1]$. Because of continuity, the above condition also holds for all $x \in [x_D - \varepsilon_D, x_D + \varepsilon_D] \subset [0, D_F^*]$, where ε_D is positive and arbitrarily small. We define X^+ and X^- as follows:

$$X^+ = \{x \notin [x_D-\varepsilon_D, x_D+\varepsilon_D], x \in [0, D_F^*] \,|\, \alpha\big[G(x;\underline{\pi}_T)-G(x;\underline{\pi}_F)\big] + (1-\alpha)[G(x;\overline{\pi}_T)-G(x;\overline{\pi}_F)] \geq 0\},$$

$$X^- = \{x \in [x_D-\varepsilon_D, x_D+\varepsilon_D] \subset [0, D_F^*] \,|\, \alpha[G(x;\underline{\pi}_T)-G(x;\underline{\pi}_F)] + (1-\alpha)[G(x;\overline{\pi}_T)-G(x;\overline{\pi}_F)] < 0\},$$

for all $\alpha \in [0,1]$. Furthermore, we define k^+ and k^- as follows:

$$k^+ = \int_{X^+}\left\{\alpha\big[G(x;\overline{\pi}_T)-G(x;\underline{\pi}_F)\big]+(1-\alpha)[G(x;\overline{\pi}_T)-G(x;\overline{\pi}_F)]\right\}dx,$$

$$k^- = \int_{X^-}\left\{\alpha\big[G(x;\underline{\pi}_T)-G(x;\underline{\pi}_F)\big]+(1-\alpha)[G(x;\overline{\pi}_T)-G(x;\overline{\pi}_F)]\right\}dx.$$

Note that $k^+ \geq 0$ and $k^- < 0$ by the definitions of X^+ and X^-. Consider an individual with utility u_D defined as

$$u''_D = \begin{cases} -1 & \text{if } x \in X^+, \\ \dfrac{k^+}{k^-} - \sigma_D & \text{if } x \in X^-, \\ 0 & \text{otherwise,} \end{cases}$$

where σ_D is a positive scalar that makes u''_D satisfy the integration by parts of FOC (8). Then, we find that

$$(1+\tau)\big(1 - G(D_F^*;\pi^*)\big)\big[1 + (1+\tau)\big(1 - G(D_F^*;\pi^*)\big)\big]$$
$$\times \left\{ \int_0^{D_F^*} u''(w - x - P(D_F^*)) \times [\alpha \left(G(x;\underline{\pi}_T) - G(x;\underline{\pi}_F)\right) + (1-\alpha)\big(G(x;\overline{\pi}_T) - G(x;\overline{\pi}_F)\big)] dx \right\}$$
$$= (1+\tau)\big(1 - G(D_F^*;\pi^*)\big)\left[-k^+ + \left(\frac{k^+}{k^-} - \sigma_D\right)k^-\right]$$
$$= (1+\tau)\big(1 - G(D_F^*;\pi^*)\big)(-\sigma_D k^-) > 0,$$

which contradicts Equation (D1).

We now prove that Condition (D2) holds for all $\alpha \in (\frac{1}{2}, 1]$ and $x \in [0, D_F^*]$ if and only if, for all $x \in [0, D_F^*]$,

$$G\big(x;\underline{\pi}_T\big) \geq G\big(x;\underline{\pi}_F\big), \tag{D3}$$

$$\big[G(x;\underline{\pi}_T) - G(x;\underline{\pi}_F)\big] + [G(x;\overline{\pi}_T) - G(x;\overline{\pi}_F)] \geq 0. \tag{D4}$$

The *if* part: Rewrite Condition (D2) as

$$(2\alpha - 1)\big[G\big(x;\underline{\pi}_T\big) - G\big(x;\underline{\pi}_F\big)\big] + (1-\alpha)\{[G(x;\underline{\pi}_T) - G(x;\underline{\pi}_F)] + [G(x;\overline{\pi}_T) - G(x;\overline{\pi}_F)]\} \geq 0. \tag{D5}$$

Note that the term $[G(x;\overline{\pi}_T) - G(x;\overline{\pi}_F)]$ can be negative. Thus, if Conditions (D3) and (D4) hold for all $x \in [0, D_F^*]$, then Condition (D5) holds for all $\alpha \in (\frac{1}{2}, 1]$ and all $x \in [0, D_F^*]$.

The *only if* part: We prove that, if there exists $x \in [0, D_F^*]$ such that $G(x;\underline{\pi}_T) < G(x;\underline{\pi}_F)$ or $\big[G(x;\underline{\pi}_T) - G(x;\underline{\pi}_F)\big] + [G(x;\overline{\pi}_T) - G(x;\overline{\pi}_F)] < 0$, then there exists $\alpha \in [\frac{1}{2}, 1]$ such that the left-hand side of Condition (D5) evaluated at x is negative. Consider the following two cases. Case 1: Suppose that there exists $x_0 \in [0, D_F^*]$ such that $G(x_0;\underline{\pi}_T) < G(x_0;\underline{\pi}_F)$. When evaluated at $\alpha = 1$, the left-hand side of Condition (D5) is negative. Case 2: Suppose that there exists $x_1 \in [0, D_F^*]$ such that $\big[G(x_1;\underline{\pi}_T) - G(x_1;\underline{\pi}_F)\big] + [G(x_1;\overline{\pi}_T) - G(x_1;\overline{\pi}_F)] < 0$. We can find an $\alpha = \frac{1}{2} + \xi$, where ξ is positive and sufficiently small so that the left-hand side of Condition (D5) is negative.

Finally, we explain the determining conditions in the two cases of the proposition. The definition of an increase in ambiguity and Assumption 3 imply that, for $u' > 0$, $G(x; \underline{\pi}_T) \leq G(x; \underline{\pi}_F) \leq G(x; \pi^*) \leq G(x; \overline{\pi}_F) \leq G(x; \overline{\pi}_T)$ for all x, with strict inequality holding for some x. That means that Condition (D3) holds at an equality for all $x \in [0, D_F^*]$. Thus, $G(x; \underline{\pi}_T) = G(x; \underline{\pi}_F)$ for all $x \in [0, D_F^*]$. Based on this result, to satisfy Assumption 2, we must have $G(x; \overline{\pi}_F) = G(x; \overline{\pi}_T)$ for all $x \in [0, D_F^*]$ to keep the center unchanged, which makes Condition (D4) hold at an equality. Thus, we have the determining conditions of Case 1, which means that $D_T^* = D_F^*$. Other things being equal, if we allow for the center to be changed after the ambiguity increase, then it can happen that $G(x; \overline{\pi}_F) \neq G(x; \overline{\pi}_T)$ for some $x \in [0, D_F^*]$. Accordingly, Condition (D4) holds for all $x \in [0, D_F^*]$, with strict inequality for some x. We now have the determining conditions of Case 2, which means that $D_T^* \leq D_F^*$.

Q.E.D.

Appendix E: Proof of Proposition 5

Assume that the SOC holds. From FOC (9), we know that $D_T^* \leq D_F^*$ if and only if

$$\int_0^{D_F^*} u'\big(w - x - P(D_F^*)\big)\, d\dot{G}_T(x;\alpha)$$
$$-\left[\frac{1}{(1+\tau)\big(1-G(D_F^*;\pi^*)\big)} - 1\right] u'\big(w - D_F^* - P(D_F^*)\big)\big[1 - \dot{G}_T(D_F^*;\alpha)\big] \leq 0. \qquad \text{(E1)}$$

The above equation can be rewritten as

$$u'\big(w - D_F^* - P(D_F^*)\big)\big\{\dot{G}_T(D_F^*,\alpha)\big[1 - \dot{G}_F(D_F^*,\alpha)\big] - \dot{G}_F(D_F^*,\alpha)\big[1 - \dot{G}_T(D_F^*,\alpha)\big]\big\}$$
$$+\int_0^{D_F^*} u''(w - x - P(D_F^*))\,\big\{\dot{G}_T(x,\alpha)\big[1 - \dot{G}_F(D_F^*,\alpha)\big] - \dot{G}_F(x,\alpha)\big[1 - \dot{G}_T(D_F^*,\alpha)\big]\big\}dx \leq 0, \qquad \text{(E2)}$$

which is obtained by multiplying FOC (9) by $\big[1 - \dot{G}_T(D_F^*,\alpha)\big]$ and Equation (E1) by $\big[1 - \dot{G}_F(D_F^*,\alpha)\big]$, subtracting the former product from the latter product, and finally integrating by parts.

Let us define $\widehat{G}_F(x,\alpha) = \dot{G}_F(D_F^*,\alpha) - \dot{G}_F(x,\alpha)$ and $\widehat{G}_T(x,\alpha) = \dot{G}_T(D_F^*,\alpha) - \dot{G}_T(x,\alpha)$ for all $x \in [0, L]$. Since $\dot{G}_i(0,\alpha) = 0$, $\widehat{G}_i(0,\alpha) = \dot{G}_i(D_F^*,\alpha)$, where $i = F$ or T. Accordingly, Equation (E2) becomes

$$u'\big(w - D_F^* - P(D_F^*)\big)\big\{\,\widehat{G}_T(0,\alpha)\big[1 - \widehat{G}_F(0,\alpha)\big] - \widehat{G}_F(0,\alpha)\big[1 - \widehat{G}_T(0,\alpha)\big]\big\}$$
$$+\int_0^{D_F^*} u''(w - x - P(D_F^*))\,\big\{\big[\widehat{G}_T(0,\alpha) - \widehat{G}_T(x,\alpha)\big]\big[1 - \widehat{G}_F(0,\alpha)\big]$$
$$-\big[\widehat{G}_F(0,\alpha) - \widehat{G}_F(x,\alpha)\big]\big[1 - \widehat{G}_T(0,\alpha)\big]\big\}dx \leq 0.$$

Furthermore, because $\int_0^{D_F^*} u''\,(w - x - P(D_F^*))\widehat{G}_i(0,\alpha)\,dx$ $= [u'(w - P(D_F^*)) - u'(w - D_F^* - P(D_F^*))]\widehat{G}_i(0,\alpha)$, where $i = F$ or T, by the definitions of $\widehat{G}_F(x,\alpha)$ and $\widehat{G}_T(x,\alpha)$, the above equation can be rearranged as

$$u'\big(w - P(D_F^*)\big)\big\{\dot{G}_T(D_F^*,\alpha)\big[1 - \dot{G}_F(D_F^*,\alpha)\big] - \dot{G}_F(D_F^*,\alpha)\big[1 - \dot{G}_T(D_F^*,\alpha)\big]\big\}$$
$$-\int_0^{D_F^*} u''\big(w - x - P(D_F^*)\big)\,\big\{\big[\dot{G}_T(D_F^*,\alpha) - \dot{G}_T(x,\alpha)\big]\big[1 - \dot{G}_F(D_F^*,\alpha)\big]$$
$$-\big[\dot{G}_F(D_F^*,\alpha) - \dot{G}_F(x,\alpha)\big]\big[1 - \dot{G}_T(D_F^*,\alpha)\big]\big\}dx \leq 0.$$

(E3)

We first prove that for all $u' > 0$, $u'' < 0$, and $\alpha \in [0,1]$, Equation (E3) holds for all $x \in [0, D_F^*]$ if and only if

$$\left[\dot{G}_T(D_F^*,\alpha) - \dot{G}_T(x,\alpha)\right]\left[1 - \dot{G}_F(D_F^*,\alpha)\right] - \left[\dot{G}_F(D_F^*,\alpha) - \dot{G}_F(x,\alpha)\right]\left[1 - \dot{G}_T(D_F^*,\alpha)\right] \leq 0, \quad \text{(E4)}$$

for all $\alpha \in [0, 1]$ and $x \in [0, D_F^*]$, as follows.

The *if* part: Since $u' > 0$ and $u'' < 0$, if Condition (E4) holds for all $\alpha \in [0,1]$ and $x \in [0, D_F^*]$, then, for all $u' > 0$, $u'' < 0$, and $\alpha \in [0,1]$, Equation (E3) holds for all $x \in [0, D_F^*]$.

The *only if* part: Suppose, by contradiction, that there exists $x_{DG} \in [0, D_F^*]$ such that

$$\left[\dot{G}_T(D_F^*,\alpha) - \dot{G}_T(x_{DG},\alpha)\right]\left[1 - \dot{G}_F(D_F^*,\alpha)\right] - \left[\dot{G}_F(D_F^*,\alpha) - \dot{G}_F(x_{DG},\alpha)\right]\left[1 - \dot{G}_T(D_F^*,\alpha)\right] > 0,$$

for all $\alpha \in [0,1]$. Due to continuity, the above equation holds for all $x \in [x_{DG} - \varepsilon_{DG}, x_{DG} + \varepsilon_{DG}] \subset [0, D_F^*]$, where ε_{DG} is positive and arbitrarily small. We define D^+ and D^- as follows:

$$D^+ = \{x \in [x_{DG} - \varepsilon_{DG}, x + \varepsilon_{DG}] \subset [0, D_F^*] \mid [\dot{G}_T(D_F^*,\alpha) - \dot{G}_T(x,\alpha)] \times [1 - \dot{G}_F(D_F^*,\alpha)] - [\dot{G}_F(D_F^*,\alpha) - \dot{G}_F(x,\alpha)][1 - \dot{G}_T(D_F^*,\alpha)] > 0\},$$

$$D^- = \{x \in [0, x_{DG} - \varepsilon_{DG}) \cup (x_{DG} + \varepsilon_{DG}, D_F^*] \mid [\dot{G}_T(D_F^*,\alpha) - \dot{G}_T(x,\alpha)] \times [1 - \dot{G}_F(D_F^*,\alpha)] - \left[\dot{G}_F(D_F^*,\alpha) - \dot{G}_F(x,\alpha)\right]\left[1 - \dot{G}_T(D_F^*,\alpha)\right] \leq 0\}.$$

In addition, we define d^+ and d^- as follows:

$$d^+ = \int_{D^+}\left\{\left[\dot{G}_T(D_F^*,\alpha) - \dot{G}_T(x,\alpha)\right]\left[1 - \dot{G}_F(D_F^*,\alpha)\right] - \left[\dot{G}_F(D_F^*,\alpha) - \dot{G}_F(x,\alpha)\right]\left[1 - \dot{G}_T(D_F^*,\alpha)\right]\right\} dx,$$

$$d^- = \int_{D^-}\left\{\left[\dot{G}_T(D_F^*,\alpha) - \dot{G}_T(x,\alpha)\right]\left[1 - \dot{G}_F(D_F^*,\alpha)\right] - \left[\dot{G}_F(D_F^*,\alpha) - \dot{G}_F(x,\alpha)\right]\left[1 - \dot{G}_T(D_F^*,\alpha)\right]\right\} dx.$$

By the definitions of D^+ and D^-, we know that $d^+ > 0$ and $d^- \leq 0$. Then, consider an individual with utility u_{DG} defined as $u'_{DG}(w - P(D_F^*)) = 0$ and

$$u''_{DG} = \begin{cases} \dfrac{d^-}{d^+} - \sigma_{DG} & \text{if } x \in D^+, \\ -1 & \text{if } x \in D^-, \\ 0 & \text{otherwise,} \end{cases}$$

where σ_{DG} is a positive scalar that makes u''_{DG} satisfy FOC (9) after integrating by parts. Because $\frac{\partial z(x,D)}{\partial x} = -1 < 0$ and $u'' < 0$, we can define $u'(w - P(D_F^*)) = 0$. Consequently,

$$
\begin{aligned}
&-\int_0^{D_F^*} u''(w-x-P(D_F^*))\left\{\left[\dot{G}_T(D_F^*,\alpha)-\dot{G}_T(x,\alpha)\right]\left[1-\dot{G}_F(D_F^*,\alpha)\right]\right.\\
&\qquad\qquad\left.-\left[\dot{G}_F(D_F^*,\alpha)-\dot{G}_F(x,\alpha)\right]\left[1-\dot{G}_T(D_F^*,\alpha)\right]\right\}dx\\
&=-\left[\left(\frac{d^-}{d^+}-\sigma_{DG}\right)d^+-d^-\right]\\
&=\sigma_{DG}d^+>0,
\end{aligned}
$$

which contradicts Equation (E3).

Next, we prove that Condition (E4) holds for all $\alpha \in (\frac{1}{2}, 1]$ and $x \in [0, D_F^*]$ through the following procedure. Let

$$
\begin{aligned}
Q(x,\alpha) = &\left[\dot{G}_T(D_F^*,\alpha)-\dot{G}_T(x,\alpha)\right]\left[1-\dot{G}_F(D_F^*,\alpha)\right]\\
&-\left[\dot{G}_F(D_F^*,\alpha)-\dot{G}_F(x,\alpha)\right]\left[1-\dot{G}_T(D_F^*,\alpha)\right].
\end{aligned}
$$

Given that $x = x_q \in [0, D_F^*]$, $Q(x_q, \alpha)$ is a quadratic function of $\alpha \in [0, 1]$. Then, by letting $\frac{dQ(x_q,\alpha)}{d\alpha} = 0$, we can find an $\alpha^*(x_q)$ at which $Q(x_q, \alpha^*(x_q))$ is a local maximum or minimum. Because the sign of $\frac{d^2Q(x_q,\alpha)}{d\alpha^2}$ cannot be determined, we consider all possible cases.

Considering the interval in which $\alpha^*(x_q)$ lies and $Q(x_q, \alpha^*(x_q))$ is a local maximum or minimum, we obtain the determining conditions under which $D_T^* \le D_F^*$ for all $u' > 0$, $u'' < 0$, and $\alpha \in (\frac{1}{2}, 1]$, as follows.

Case 1. $\frac{d^2Q(x_q,\alpha)}{d\alpha^2} \le 0$ at $\alpha^*(x_q) \in [0, \frac{1}{2}]$.

Since $Q(x_q, \alpha^*(x_q))$ is a local maximum when $\alpha^*(x_q)$ lies in $[0, \frac{1}{2}]$, the condition $Q(x_q, \alpha) \le 0$ for all $\alpha \in (\frac{1}{2}, 1]$ is equivalent to requiring $Q(x_q, \frac{1}{2} + \varepsilon) \le 0$ for $0 < \varepsilon \le \frac{1}{2}$.

Case 2. $\frac{d^2Q(x_q,\alpha)}{d\alpha^2} \ge 0$ at $\alpha^*(x_q) \in [0, \frac{1}{2}]$.

Since $Q(x_q, \alpha^*(x_q))$ is a local minimum when $\alpha^*(x_q)$ lies in $[0, \frac{1}{2}]$, the condition $Q(x_q, \alpha) \le 0$ for all $\alpha \in (\frac{1}{2}, 1]$ is equivalent to requiring $Q(x_q, 1) \le 0$.

Case 3. $\frac{d^2Q(x_q,\alpha)}{d\alpha^2} \le 0$ at $\alpha^*(x_q) \in (\frac{1}{2}, 1]$.

Since $Q(x_q, \alpha^*(x_q))$ is a local maximum when $\alpha^*(x_q) \in (\frac{1}{2}, 1]$, the condition $Q(x_q, \alpha) \le 0$ for all $\alpha \in (\frac{1}{2}, 1]$ is equivalent to requiring $Q(x_q, \alpha^*(x_q)) \le 0$.

Case 4. $\frac{d^2Q(x_q,\alpha)}{d\alpha^2} \ge 0$ at $\alpha^*(x_q) \in (\frac{1}{2}, 1)$.

Since $Q(x_q, \alpha^*(x_q))$ is a local minimum when $\alpha^*(x_q)$ lies in $(\frac{1}{2}, 1)$, the condition $Q(x_q, \alpha) \le 0$ for all $\alpha \in (\frac{1}{2}, 1]$ is equivalent to requiring $Q(x_q, \frac{1}{2} + \varepsilon) \le 0$ and $Q(x_q, 1) \le 0$, where $0 < \varepsilon \le \frac{1}{2}$.

Case 5. $\frac{d^2Q(x_q,\alpha)}{d\alpha^2} \ge 0$ at $\alpha^*(x_q) = 1$.

Since $Q(x_q, \alpha^*(x_q))$ is a local minimum when $\alpha^*(x_q) = 1$, the condition $Q(x_q, \alpha) \leq 0$ for all $\alpha \in (\frac{1}{2}, 1]$ is equivalent to requiring $Q(x_q, \frac{1}{2} + \varepsilon) \leq 0$, where $0 < \varepsilon \leq \frac{1}{2}$.

Q.E.D

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