# Market Liquidity and Trade Reactions to Accounting Disclosures

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## **Abstract**

The analysis of capital markets generally depends on assumptions about the structure of market information and about how traders process information. The various equilibrium paradigms used in the research on asset market behavior differ in their assumptions with regard to the amount of information conveyed in price and the information sets used by traders for their portfolio decisions. This paper analyzes market responses to accounting disclosures with a two-period (three-date) noisy rational expectations model. There are three types of risk-neutral agents: a market maker, informed traders, and liquidity traders. The informed traders receive private signals and the firm releases an accounting report at the first and second dates, respectively. Our model considers two settings where the sequence of prices can either fully reveal or partially reveal private signals. We investigate trading volume responses to a financial accounting disclosure at the time of announcement under these frameworks. Furthermore, we examine how the level of information asymmetry and the degree of liquidity affect the magnitude of trading volume reaction.

Conclusions of this paper are as follows. First, if the private signals are not fully revealed in the sequence of prices, trade in the risky asset occurs at the second date with trading volume arising from both informed and liquidity trading. Second, no informed trading takes place for the risky asset at the second date when

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the private signal is fully revealed by either a public announcement or the price sequence. This implies that no informed traders submit orders for the risky asset at date 2 and all demand orders for the risky asset are from liquidity traders. Third, in the setting of partially revealing private signal, market liquidity at date 2 is increasing in both the precision of a public announcement and the number of liquidity traders, and decreasing in the diversity of opinion among informed traders.

**Keywords:** Public Disclosures, Information Asymmetry, Market Liquidity, Noisy Rational Expectations Model

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## Introduction

The purpose of this paper is to examine how the information content of a financial accounting disclosure influences the reaction of market participants as well as how the sequence of prices affects traders' trading incentive by partially, or fully revealing to market makers the information known to informed traders. The analysis is based on a two-period (three-date) noisy rational expectations model. Prior studies of this issue, such as Diamond (1985), Bushman (1991), and Lundholm (1991), have assumed that a public report release and private information acquisition occur simultaneously. As a result, a public disclosure serves as a substitute for a private signal and generally reduces traders' incentive to invest in information acquisition. Brown and Jennings (1989)<sup>1</sup> and Lin and Wang (2001) examine a two-period setting in which each trader receives a private signal, possibly different, in each period before he trades. In contrast, in the economy Grundy and McNichols (1989) consider, the traders are allowed to acquire and trade on a private signal prior to a public disclosure. Because a forthcoming public disclosure can stimulate private signal collection in the pre-announcement period, they obtain two types of multiperiod noisy rational expectations equilibrium models. In the first, the informed traders do not learn about the average private signal at the second trading date and therefore no informed trading take place for the risky asset at that date. In the second, the informed traders receive the average private signal at the second date and trade thus occurs at both trading dates. This paper uses a framework similar to that of Grundy and McNichols, where traders receive private signals at the first trading date and a public signal is disclosed at the second date.

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<sup>&</sup>lt;sup>1</sup> Brown and Jennings (1989) show that technical analysis has value in a two-period model in which traders have rational expectations about the relation between signals and prices. However, they do not consider how information revealed by the price sequence affects trading volume.

Recent models by Lin, Wang and Tsai (1995), Demski and Feltham (1994), Kim and Verrecchia (1994), and McNichols and Trueman (1994), also examine the impact of a public disclosure on private information collection in multiperiod rational expectations economies. The first two papers assume that all traders are risk-averse and set their demand for the firm's shares believing that it does not affect the firm's market price. On the other hand, the second two papers assume that the informed trader is risk-neutral and takes into account how his demand for a firm's share affects the market price. We follow a setting similar to that of the second two papers, where traders are risk-neutral.<sup>2</sup> Kim and Verrecchia (1994) suggest a model of trade in which financial accounting disclosures simultaneously induce increased information asymmetry, less market liquidity, and more trading volume.<sup>3</sup> There are four types of risk-neutral agents in their economy: a market maker, potential information processors, nondiscretionary liquidity traders, and discretionary liquidity traders. Lin and Wang (2001) assume there are both informed and noise traders in the market, and investigate the patterns of trading volume in a setting where pure noise trading volume is correlated or uncorrelated with observable market variables, such as prices and public releases.<sup>4</sup> Our paper,

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<sup>&</sup>lt;sup>2</sup> By assuming risk neutrality, we can focus on analyzing the impacts of private information and public disclosure on the informed trader's expected profit and market liquidity.

<sup>&</sup>lt;sup>3</sup> Kim and Verrecchia (1994) suggest that there may be more information asymmetry at the time of an announcement than in nonannouncement periods because an earnings announcement allows certain traders to make judgments about a firm's performance that are superior to the judgments of other traders. More information asymmetry implies that bid-ask spread increase, suggesting that market liquidity decrease.

Their results indicate that when pure noise trading volume is uncorrelated with observable market variables, no informed trading occurs with constant net supply. And when net supply is with random shock, the time 2 holding units of the risky asset by informed and noise traders are different from the time 1 holding units, but only the informed trading volume is predictable. In the case of pure noise trading volume being correlated with observable market variables, the informed traders also do not trade when there is no supply shock. However, when net supply contains random shocks, trading volume consists of noise and informed trading, both of which can be estimated.

on the other hand, considers three types of traders: informed traders, who choose the precision of their private signals; liquidity traders, whose demands for the risky asset are unrelated to any information in the market; and a competitive market maker, who establishes prices. In this setting, we examine the effect of a public signal on the informed and the liquidity trading, as well as on the private information acquisition activities in an intuitive way.

McNichols and Trueman (1994) show how public disclosures would affect prior information acquisition activities and pre-announcement security prices. For convenience, they employ Kyle (1985)'s framework, where there is only a single informed trader. This simplifying assumption is not plausible on empirical grounds. Our model allows for multiple informed traders and therefore is able to reveal empirical implications. Furthermore, Kim and Verrecchia (1994) investigate the effects of a public disclosure, private information and the firm's liquidating value before earnings announcement date on the expected profit of an information processor at earnings announcement date. In their analysis, private information alone is not an informative signal about the firm's liquidating value as the dissemination of a public signal occurs simultaneously with informed traders' private signal acquisition. Different from their analysis, this paper adopts a two-trading-date model, where the private signal is acquired prior to the first trading date and a public report is released between the first and second trading dates. Because the central feature of this analysis is the impact of exogenous change on the information content, our model consider two types of equilibria. In the first type of equilibrium, the private signal is partially revealed by the price sequence, whereas the private signal is fully revealed by the price sequence or public disclosures in the second type of equilibrium. Under these two frameworks, we examine the price change of and the traders' demand for the risky asset during the two trading dates. Also, we investigate whether a public information release accompanies the opportunity to retrade.

Related theoretical literature pays little explicit attention to the impacts of public disclosures on market liquidity. Disclosures generally contain two salient properties. The first property is that they disseminate data of which there may be no alternative sources. The second property comes from the fact that they provide information, which may induce different interpretations about the firm's performance. Public disclosures broadly include earnings announcements, analysts' forecasts, and other summaries of detailed financial accounting data. Indeed, there are two ways to describe public disclosures, and each of them has different empirical intuition. First, Kim and Verrecchia (1994) indicate that some market participants process earnings announcements into private, and possibly diverse, information about the firm's performance. This private information can be viewed as informed judgment of traders.<sup>5</sup> If there are no announcements, traders cannot obtain informed opinions through their ability to process public signals. In other words, public announcements motivate informed judgments among traders who process public releases into private signals, and thus exacerbate information asymmetries between them and the market maker. The discretionary liquidity traders will avoid trading and thus trading volume only comes from informed traders. 6 It follows that the market becomes less liquid. 7

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<sup>&</sup>lt;sup>5</sup> The empirical study of Lys and Shon (1990) supports this issue. They suggest that analysts process public disclosures into private informed judgment.

Admati and Pfleiderer (1988) indicate that the discretionary liquidity traders trade in the greatest liquidity periods, which may lead to a positive association between trading volume and market liquidity. However, their setting does not consider public disclosures.

<sup>&</sup>lt;sup>7</sup> Less liquidity does not imply less trading activity around public announcements. At this time, discretionary liquidity traders will avoid trade. They choose between being relatively informed and trading in relatively illiquid markets versus being relatively uninformed and trading in liquid markets. In other words, as the market maker does not analyze in great detail the information content of a disclosure, earnings announcements provide information processors with a temporary advantage about the firm's performance assessments over the market maker. When the information processors are significantly active, more trading volume may also result despite the reduction in liquidity.

An alternative description of public disclosures can be obtained by considering a setting in which the informed traders own superior information about the firm's performance based on their personal affiliation. Because public disclosures can partially or fully reveal informed traders' information to the market maker, the adverse selection problems will lessen. This implies that a disclosure makes the prices quoted by the market maker less sensitive to buy and sell orders and market becomes more liquid around earnings announcements. The intuition is that the market marker sets a low bid-ask spread when an announcement occurs and reduces information asymmetry. Therefore, our paper also examines the effect of public disclosures on information asymmetry and market liquidity, with an aim to find out which of the two descriptions can better fit our model.<sup>8</sup>

The conclusions of this paper are as follows. If the private signal is partially revealed by the sequence of prices, trade occurs for the risky asset at the second date where trading volume results from both informed and liquidity trading. However, no informed trading takes place for the risky asset at the second date when the private signal is fully revealed by either a public disclosure or the price sequence. This situation is equivalent to no private information in the market, and consequently all demand orders for the risky asset are from liquidity traders at the time of a public announcement. Moreover, because trading volume is the change in unit holdings for the risky asset during two dates, the analysis of market liquidity is more meaningful at the second trading date. Our result indicate that in the setting of price sequence not fully revealing private signal, market liquidity at date 2 is increasing in both the precision of a public signal and the number of liquidity traders, and decreasing in the diversity of opinion among informed traders. These

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<sup>&</sup>lt;sup>8</sup> Grundy and McNichols model only considers rational traders, while our paper discusses informed traders, liquidity traders, and the market maker. Also, they don't demonstrate the effects that public disclosures have on information asymmetry and market liquidity.

results are obtained because the market maker sets a low bid-ask spread when an announcement occurs that reduces information asymmetry and increases market liquid. Although our results are similar to Diamond and Verrecchia (1991), and Kim and Verrecchia (1994), our model highlights different perspectives from their models. We further examine the change in security price and the pattern of trading volume when private signals are fully revealed or partially revealed by the sequence of prices or a public disclosure in a multiperiod rational expectations equilibrium model. Hence, our setting is more consistent with economic intuition than the previous studies.

The layout of the paper goes as follows. Section 2 describes a detailed description of our model. Section 3 characterizes an equilibrium and market behavior statistics in a market comprised of a market maker, informed traders, and liquidity traders. Section 4 analyzes market liquidity and trading volume at the time of a public disclosure. Comparative static is also developed in Section 4. Conclusions and suggestions for future research are contained in Section 5.

## The basic model

The basic framework of our analysis follows Grundy and McNichols (1989) and Kim and Verrecchia (1994). The analysis is based on a three-date (two-period) noisy rational expectations model. The endogenous private signal collection and trading happen at the first date, a public information release and trading take place at the second date, and the return of the risky asset is realized and consumption occurs at the third date. We consider two types of equilibria; the private signals are either fully revealed or partially revealed by the price sequence or a public disclosure. The pattern of trading volume and the change in prices are analyzed

under these two equilibria. Furthermore, the effect of financial accounting disclosures on market liquidity and information asymmetry is also discussed in our model.

Assume that in a pure exchange economy, the traders allocate their wealth between one risky asset and riskless assets in periods 1 and 2. One unit of riskless assets pays off one unit of consumption good at date 3. Without loss of generality, the riskless rate of interest is assumed to be zero. The return of the risky asset is a random variable, denoted by  $\widetilde{F}$ , and is realized at date 3. It is assumed that  $\widetilde{F}$  is normally distributed with mean 0 and variance  $\sigma_F^2$ . There are three types of risk-neutral agents in this economy: a market maker, informed traders, and liquidity traders. A set of N informed traders are indexed by i=1,2,...,N. Informed trader i's holding of the risky asset in each period, denoted by  $\widetilde{x}_{ii}$ , t=1,2, is chosen to maximize the expected profits conditional on his available information. The aggregate demand of informed traders is denoted by  $\widetilde{X}_{II}$ , where  $\widetilde{X}_{II} = \sum_{i=1}^{N} \widetilde{x}_{II}$ . The liquidity traders are the second group of traders. Our model is established with an

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Suppose that R is the return on the riskless asset, it is equal to 1 plus the riskless rate of interest. If the riskless rate of interest is assumed to 0, it implies the price of the riskless asset, R, is equal to unity. Because R is constant and not a random variable, when the riskless rate of interest is not zero ( $R \ne 1$ ), it doesn't influence the solution process of trader i's demand for the risky asset at dates 1 and 2,  $\widetilde{x}_{i1}$  and  $\widetilde{x}_{i2}$ . Also, as consumption occurs only at the final date, traders' marginal utilities at dates 1 and 2 are indeterminate without an exogenous specification of the riskless rate.

<sup>&</sup>lt;sup>10</sup> It is assumed without loss of generality that all random variables have zero mean.

When the precision of a private signal is higher, it involves more processing cost. The choice benchmark of informed traders is for the marginal benefit to be equal to the marginal cost. Because our model considers multiple informed traders rather than a single informed trader, informed traders' choice of the precision is a game theory problem. This issue will increase the complication of our paper. Our main purpose is to examine the effects of a public signal on the informed and the liquidity trading, as well as on information asymmetry and market liquidity. We also discuss the change in security price and the pattern of trading volume in a setting where private signals are fully revealed or partially revealed by the sequence of prices or by a public disclosure. In the future research, this model can be extended to the problem of informed traders' choice of the precision. About this suggestion, we would like to thank the referee.

exogenously given number of liquidity traders, L. We assume that a liquidity trader h's net demand for shares at date t,  $\tilde{x}_{ht}$ , is a normally distributed random variable with mean 0 and variance 1. The liquidity trader's decision is unrelated to any information in the market. The aggregate demand of liquidity traders is denoted by  $\widetilde{X}_{Lt}$ , where  $\widetilde{X}_{Lt} = \sum_{h=1}^{L} \widetilde{x}_{ht}$ . In the following section, we derive the market equilibrium with an exogenously given number of informed traders, N, and a given variance of  $\widetilde{X}_{Lt}$ ,  $\ell$ . Denote by  $\widetilde{X}_{t}$  (t=1,2) the total market demand order at time t, where  $\widetilde{X}_t = \sum_{i=1}^{N} \widetilde{x}_{it} + \sum_{h=1}^{L} \widetilde{x}_{ht}$ . The third type of traders is the market maker; he earns zero profits at each period. Both informed and liquidity traders submit their market orders for the risky asset to the market maker. The market maker offers shares for sale so as to clear the market at that time. Also, the market maker cannot distinguish among orders from different types of agents, but can infer the private signal from the total market demand order  $\widetilde{X}_t$ . Therefore, acting as a perfect competitor, the market maker at date 2 sets price  $\widetilde{P}_2$  equal to his expectation of the terminal value  $\widetilde{F}$ , conditional on his information of a public disclosure  $\widetilde{Z}$  and total demand  $\widetilde{X}_2$ . Similarly, the market maker offers a price  $\widetilde{P}_1$  at date 1, based on observation of total market order  $\widetilde{X}_1$ . Moreover, informed trader i chooses demand given the market maker's pricing strategy.

Informed trader i privately receives imperfect information about the terminal value (at date 3) of the risky asset before he makes trading decisions. At the beginning of period 1, informed trader i observes a private signal  $\widetilde{Y}_i$  of the form

$$\widetilde{Y}_{i} = \widetilde{F} + \widetilde{e}_{i}$$

<sup>12</sup> This zero-profit assumption is commonly used to reflect the fact that market making is competitive and closely regulated. See Kim and Verrecchia (1994).

where an idiosyncratic noise term,  $\widetilde{e}_i$ , is a normally distributed random variable that is uncorrelated with  $\widetilde{F}$ , and has a mean of zero and variance of  $\sigma_e^2$  for all i. The covariance between the error terms  $\widetilde{e}_i$  and  $\widetilde{e}_j$ , for any two informed traders i and j, is  $\rho_e^2$  where  $\rho_e^2$  is the correlation coefficient and is assumed to lie between (and including) 0 and 1. If  $\rho_e^2$  1, the error terms are identical and all informed traders observe the same information. In contrast, the error terms among different informed traders are independent of each other when  $\rho_e^2$  0. Hence, in our model, diversity of opinion among informed traders is measured by 1- $\rho_e$ . To keep the facility of analysis, we assume that the informed traders have homogeneous priors, whereas posteriors can be either homogeneous or heterogeneous, depending on  $\rho_e$ .

We assume a public announcement to be a competing source of information at date 2, and all traders observe it. The publicly announced signal  $\widetilde{Z}$  is a noisy measure of the risky asset's terminal value of the form

$$\widetilde{Z} = \widetilde{F} + \widetilde{V}$$

where a common noise term,  $\widetilde{v}$ , is an independent, normally distributed random variable with mean 0 and variance  $\sigma_v^2$ . Suppose that  $\widetilde{v}$  is independent of  $\widetilde{F}$ ,  $\widetilde{X}$  and  $\widetilde{e}_i^{-13}$  We seek an equilibrium in which the market participants make conjectures about the actions of others and the conjectures turn out to be correct. Our model considers two settings in the next section. In the first setting, the date 2 public disclosure  $\widetilde{Z}$  is not sufficient to reveal the date 1 private signal  $\widetilde{Y}_i^{-14}$  and the market maker sets  $\widetilde{P}_2$  conditional on the public disclosure and the observable aggregate demand  $\widetilde{X}_2$ . This setting implies the information content of a

<sup>13</sup> All random variables in our model are mutually independent unless otherwise assumed.

<sup>&</sup>lt;sup>14</sup> This reflects a setting in which the informed traders are attempting to forecast a forthcoming public disclosure.

forthcoming accounting earnings contained in the time 1's private signal has only been partially impounded in both times' prices. In the second setting, the price sequence can fully reveal the date 1 private signal when a public announcement is a sufficient statistic for the private information. As a result, informed trader i's demand for the risky asset at date 2 is only determined by a public disclosure. The market maker sets  $\widetilde{P}_2$  equal to his expectation of  $\widetilde{F}$  conditional on a public disclosure. The empirical implication is that if an announcement fully contains the information of the date 1 private signal, the informed traders base their decisions making on accounting disclosure. This reflects the fact that the trade incentive of informed traders is mainly from private information.

# Market equilibrium

In this section we derive the market equilibrium with an exogenously given number of informed traders, N, and a given variance of liquidity trading,  $\ell$ . We first demonstrate the properties under the setting of the price sequence unable to fully reveal the private signal of the date 1. Second, we relax the above assumption and discuss the properties when the price sequence is able to fully reveal the date 1 private signal. Because the informed traders are risk-neutral, their respective demand is independent of the endowment of the risky asset. This implies that trading volume, market liquidity, and price are also unrelated to the endowment of the risky asset.

At date 1, the informed traders choose their demand orders for the risky asset,  $\tilde{x}_{i1}$ , based on their private information. The date 1 demand of informed trader

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<sup>&</sup>lt;sup>15</sup> We ignore it without loss of generality throughout this paper.

i can be expressed as

$$\widetilde{x}_{i1} = k\widetilde{Y}_i \tag{1}$$

At date 2, a public disclosure about the firm's value is released before the informed traders close out their position. Informed trader i's demand for the risky asset is related to the date 1 private signal and the date 2 public disclosure, and it can be written as  $^{16}$ 

$$\widetilde{x}_{i2} = \theta_1 \widetilde{Y}_i + \theta_2 \widetilde{Z} \tag{2}$$

Furthermore, at date 2, the market maker sets  $\widetilde{P}_2$  equal to his expectation of the firm's liquidating value  $\widetilde{F}$ , conditional on the public disclosure and his observation of the date 2 order flow. That is, the date 2 price is a linear function of the public disclosure and the aggregate supply for the risky asset. We can then express the date 2 price as  $\widetilde{P}_2 = E(\widetilde{F} | \widetilde{X}_2, \widetilde{Z}) = b_1 \widetilde{X}_2 + b_2 \widetilde{Z}$ . Because the price conjecture is a linear function of normal variables, it is normally distributed. Using the backward solution technique of the dynamic programming, we first solve the equilibrium price of and demand (holding units) for the risky asset in the second period. Secondly, we solve the date 1 equilibrium price and demand of informed traders for the risky asset.

At date 2, informed trader i trades given his available information to maximize his expected profits. Therefore, given the market maker's pricing

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The informed trader chooses demand for the risky asset given the market maker's pricing strategy.  $\widetilde{P}_2$  ( $\widetilde{P}_1$ ) is the price of the firm set by the market maker at date 2 (date 1). The market maker sets  $\widetilde{P}_1$  equal to his expectation of the liquidating value  $\widetilde{F}$ , given observation of total market order. Similarly, the market maker sets  $\widetilde{P}_2$  conditional on total market order and a public signal  $\widetilde{Z}$ , if released. Because the informed trader is risk neutral and the market maker sets price, informed trader i's demand order for shares is unrelated to the market price.

(4)

strategy, informed trader *i* chooses  $\tilde{x}_{i2}$  to maximize

$$\max_{\widetilde{X}_{i2}} E\left[\widetilde{X}_{i2}(\widetilde{F} - \widetilde{P}_{2}) \middle| \widetilde{Y}_{i}, \widetilde{Z}\right]$$

By solving the first-order condition the informed trader's optimal date 2 holdings in the risky asset are given by (see Appendix 1 for details):

$$\widetilde{x}_{i2} = \frac{1}{2b_{1}} \left\{ \left[ \frac{\sigma_{F}^{2} \sigma_{v}^{2} - (N-) b_{1} \theta_{1} \left[ \sigma_{v}^{2} \sigma_{F}^{2} + \rho_{e}^{2} \left( \sigma_{F}^{2} + \sigma_{v}^{2} \right) \right]}{\sigma_{F}^{2} \sigma_{e}^{2} + \sigma_{F}^{2} \sigma_{v}^{2} + \sigma_{e}^{2} \sigma_{v}^{2}} \right] \widetilde{Y}_{i} 
+ \left[ -\left[ b_{2} + \theta_{2} b_{1} (N-) \right] + \frac{\sigma_{F}^{2} \sigma_{e}^{2} - (N-) b_{1} \theta_{1} 1 - \rho_{F}^{2} \sigma_{e}^{2} \sigma_{F}^{2}}{\sigma_{F}^{2} \sigma_{e}^{2} + \sigma_{F}^{2} \sigma_{v}^{2} + \sigma_{e}^{2} \sigma_{v}^{2}} \right] \widetilde{Z} \right\}$$
(3)

where

$$\theta_{1} = \frac{\sigma_{F}^{2} \sigma_{v}^{2}}{b_{1} \{ (N+) \sigma_{F}^{2} \sigma_{v}^{2} + [2 + N + 1 \rho] \sigma_{e}^{2} (\sigma_{F}^{2} + \sigma_{v}^{2}) \}}$$

$$\theta_{2} = \frac{1}{(N+1)b_{1}} \left\{ -b_{2} + \frac{\sigma_{F}^{2} \sigma_{e}^{2} [2 + (N-)] \beta}{(N+) \sigma_{F}^{2} \sigma_{v}^{2} + [2 + N+1] \beta \sigma_{e}^{2} (\sigma_{F}^{2} + \sigma_{v}^{2})} \right\}$$

The parameters  $\theta_1$  and  $\theta_2$  are shown in the Appendix 2. The following proposition provides a characterization of the equilibrium.

**Proposition 1.** For the case where informed trader i receives a private signal at date 1 and a public disclosure occurs at date 2, partially revealing rational expectations equilibrium exists in which  $\widetilde{P}_2$ , the date 2 equilibrium price is given by (see Appendix 3 for details)

$$\tilde{P}_2 = E\left[\tilde{F}\middle|\tilde{X}_2,\tilde{Z}\right] = b_1\tilde{X}_2 + b_2\tilde{Z}$$

where

$$b_{1} = \sqrt{\frac{N\sigma_{F}^{4}\sigma_{v}^{4}\left(\sigma_{F}^{2}\sigma_{v}^{2} + \sigma_{F}^{2}\sigma_{e}^{2} + \sigma_{e}^{2}\sigma_{v}^{2}\right)}{\ell\left(\sigma_{F}^{2} + \sigma_{v}^{2}\right)\left\{\left(N + \right)\sigma_{F}^{2}\sigma_{v}^{2} + \left[2 + N + 1\rho\right]\sigma_{e}^{2}\left(\sigma_{F}^{2} + \sigma_{v}^{2}\right)^{2}\right\}^{2}}}$$

$$b_{2} = \frac{\sigma_{F}^{2} \left[ (N - ) \left( \sigma_{F}^{4} - 2\sigma_{F}^{2} \sigma_{e}^{2} \right) + 2\sigma_{e}^{2} \sigma_{v}^{2} (N + ) + \sigma_{F}^{2} \sigma_{v}^{2} N - \left( \sigma_{F}^{2} + \sigma_{v}^{2} \right) \sigma_{e}^{2} \rho (N - )^{2} \right]}{\left[ \left( \sigma_{F}^{2} + \sigma_{v}^{2} \right) \left[ \left( 1 - \rho \right) \left( \sigma_{v}^{2} \sigma_{e}^{2} - 2N\sigma_{e}^{2} \sigma_{v}^{2} + \sigma_{e}^{2} \sigma_{F}^{2} \right) - N^{2} \rho \left( \sigma_{F}^{2} + \sigma_{v}^{2} \right) + \left( \sigma_{v}^{2} + \sigma_{F}^{2} \sigma_{F}^{2} + \sigma_{F}^{2} \sigma_{e}^{2} (1 - 2N) \right) \right]}$$

In the meantime, informed trader i 's demand for shares at date 2 is given by (see Appendix 4 for details)

$$\widetilde{x}_{i2} = \frac{1}{2b_{1}} \left\{ \left[ \frac{\sigma_{F}^{2} \sigma_{v}^{2} - (N-) b_{1} \theta_{1} \left[ \sigma_{v}^{2} \sigma_{F}^{2} + \rho_{e}^{-2} \left( \sigma_{F}^{2} + \sigma_{v}^{2} \right) \right]}{\sigma_{F}^{2} \sigma_{e}^{2} + \sigma_{F}^{2} \sigma_{v}^{2} + \sigma_{e}^{2} \sigma_{v}^{2}} \right] \widetilde{Y}_{i}$$

$$+ \left[ -\left[ b_{2} + \theta_{2} b_{1} (N-) \right] + \frac{\sigma_{F}^{2} \sigma_{e}^{2} - (N-) b_{1} \theta_{1} 1 - \rho_{F}^{2} \sigma_{e}^{2} \sigma_{F}^{2}}{\sigma_{F}^{2} \sigma_{e}^{2} + \sigma_{F}^{2} \sigma_{v}^{2} + \sigma_{e}^{2} \sigma_{v}^{2}} \right] \widetilde{Z} \right\}$$
(5)

where 
$$\theta_1 = \sqrt{\frac{\ell(\sigma_F^2 + \sigma_v^2)}{N(\sigma_F^2 \sigma_v^2 + \sigma_F^2 \sigma_e^2 + \sigma_e^2 \sigma_v^2)}}$$

$$\theta_2 = \frac{\mathbf{E}}{\mathbf{F}}$$
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Eqs. (4) and (5) indicate a complete characterization of the unique market equilibrium at the time of a public disclosure for any given numbers of the informed and liquidity traders. Now step backward to the trader's allocation problem at date 1. At date 1, informed trader i trades conditional on his private information. That is, the date 1 holdings arise as the solution to informed trader i's optimization problem

$$\max_{\tilde{X}_{i1}} E \left[ \tilde{x}_{i1} (\tilde{P}_2 - \tilde{P}_1) \middle| \tilde{Y}_i \right]$$

As discussed in the previous section, the market maker at date 1 sets  $\widetilde{P}_1$  equal to his expectation of the firm's liquidating value  $\widetilde{F}$ , given observation of the date 1 aggregate order flow. Thus,  $\widetilde{P}_1$  can be written as  $\widetilde{P}_1 = E\left[\widetilde{F} \middle| \widetilde{X}_1 \right] = a\widetilde{X}_1$ . By solving the first-order condition the optimal demand of informed trader i is obtained as follows. The equilibrium properties of informed trader i at date 1 are captured by the following proposition.

**Proposition 2.** For the case where informed trader i acquires private information at date 1, partially revealing rational expectations equilibrium exists in which  $\tilde{x}_{i1}$ , the demand for shares by informed trader i, is given by

$$\widetilde{x}_{i1} = k \widetilde{Y}_i \tag{6}$$

where

$$k = \frac{1}{2a} \left\{ b_1 \theta_1 + \left( b_1 \theta_1 - a \mathbf{k} \right) N + 1 \frac{\sigma_F^2 + \boldsymbol{\rho}_e^2}{\sigma_F^2 + \sigma_e^2} + \left( b_1 N \theta_2 + b_2 \right) \frac{\sigma_F^2}{\sigma_F^2 + \sigma_e^2} \right\}$$

In addition, the date 1 equilibrium price is (see Appendix 5 for details)

$$\widetilde{P}_1 = a\widetilde{X}_1 \tag{7}$$

where

$$a = \frac{Nk\sigma_F^2}{\ell + Nk^2(\sigma_F^2 + \sigma_e^2) + k^2N(N-)(\sigma_F^2 + \sigma_e^2)}$$
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Eqs. (6) and (7) indicate a complete characterization of the unique market

equilibrium at date 1 for any given numbers of the informed and liquidity traders. 17

In the above discussion, we assume that informed trader i observes the private signal,  $\widetilde{Y}_i$ , at date 1 and a public disclosure,  $\widetilde{Z}$ , at date 2. Our model describes the equilibrium where the sequence of prices or a public disclosure cannot fully reveal private signals. In that case, informed trader i's demand for the risky asset at date 2 is also influenced by the private signal. That is, the date 1 private signal is valuable for informed traders' decision making at date 2. Now suppose that the sequence of prices fully reveals the private signal. Then a public disclosure,  $\widetilde{Z}$ , at date 2 is a sufficient statistic for  $(\widetilde{Z},\widetilde{Y})$ , where  $\widetilde{Y}_i$  is the private signal at date 1. This implies that  $\widetilde{Y}_i$  is redundant for decision making after traders received  $\widetilde{Z}$ .

In this reduced form, the information content of a public announcement is more than or equal to that of private signals. This is because the common error of a public disclosure,  $\widetilde{V}$ , contains the idiosyncratic noise term of private signals,  $\widetilde{e_i}$ . In the above discussion, informed trader i's demand for the risky asset at date 2 is a linear function of the private information and a public disclosure because the price sequence cannot fully reveal private signals. Also, at date 2, the price quotations of the market maker are based on a public disclosure and observable market orders for the risky asset. We now assume that  $\widetilde{Z}$  is a sufficient statistic for  $(\widetilde{Z},\widetilde{Y})$ . As a result, informed trader i's demand for the risky asset at date 2 is only determined by a public announcement and the market maker sets  $\widetilde{P}_2$  equal to his expectation of  $\widetilde{F}$  conditional on a public signal. This reflects the fact that the informed

In the case of partially revealing, when the coefficients k,  $b_1$ ,  $b_2$ ,  $\theta_1$ , and  $\theta_2$  are solved in eqs. (6) and (7), we can find that the demand of informed trader i at date 1,  $\widetilde{x}_{i1}$ , is unrelated to the precision of the time 2 public disclosure (or the variance of common noise term of a public disclosure,  $\sigma_v^2$ ). The intuition behind this is that our paper assumes that the informed traders are risk neutral, and they are not concerned about the variance of common error of a public disclosure.

We assume that the following relation holds  $\widetilde{Z} = \widetilde{F} + \widetilde{v} + \widetilde{e}$ , where  $\widetilde{e} = \sum_{i=1}^{N} \widetilde{e}_i$ .

In this case,  $\tilde{P}_2 = E\left[\tilde{F} \middle| \tilde{X}_2, \tilde{Z} \middle] = h_1 \tilde{X}_2 + h_2 \tilde{Z}$ , the coefficient  $h_1$  can be solved and it equals zero. Consequently,  $\tilde{P}_2$  can be simplified as  $\tilde{P}_2 = h_2 \tilde{Z}$ . That is, the price quoted by the market maker is based on a public disclosure.

traders only trade for the riskless assets at date 2 because there is no private information in the market. These properties are formalized in Proposition 3.

**Proposition 3.** If informed trader *i* observes the private signal at date 1 and a public report at date 2, fully revealling rational expectations equilibrium exists in which

- (1) The date 2 equilibrium price,  $\widetilde{P}_2$ , is a sufficient statistic for all traders' private information and each informed trader does not submit market orders for the risky asset at date 2.
- (2) The informed trader i 's demand for shares at date 1,  $\tilde{x}_{i1}$ , is given by (see Appendix 6 for details)

where 
$$k = \sqrt{\frac{\sigma_F^2 \ell}{G(\sigma_F^2 + \sigma_v^2 + \sigma_v^2)}}$$

(3)  $\widetilde{P}_1$ , the date 1 equilibrium price is given by ( see Appendix 6 for details )

$$\widetilde{P}_{1} = a\widetilde{X}_{1} \tag{9}$$

where

$$a = \frac{Nk\sigma_F^2}{\ell + Nk^2(\sigma_F^2 + \sigma_e^2) + k^2N(N-)(\sigma_F^2 + \sigma_e^2)}$$
#

Hence, if the price sequence or a public disclosure fully reveals the private signal, the informed traders do not trade for the risky asset at date 2. This implies that at date 2, private information is not valuable in the market, and consequently the informed traders do not change the holding units of the risky asset. Note, the

informed traders observe the private signal,  $\widetilde{Y}_i$ , at date 1 and a public disclosure,  $\widetilde{Z}$ , at date 2. The variance of the common noise term of a publicly announced signal,  $\sigma_v^2$ , is unrelated to informed trader i's holding units of the risky asset at date 1,  $\widetilde{x}_{i1}$ .<sup>20</sup>

# Market liquidity and trading volume

In this section we analyze whether a public disclosure creates information asymmetries between the market maker and the informed traders. Information asymmetry influences market liquidity, which, in turn, affects other aspects of the analysis (for example, volume). As assumed in the previous section, the number of informed traders at the time of a public disclosure, N, is exogenously determined. Also, our model is established with an exogenously given number of liquidity traders, L. The number of informed and liquidity traders remain constant in both periods. Therefore, we will provide a characterization of the market equilibrium around a public disclosure for any given number of informed and liquidity traders.

Because the informed traders do not submit orders for the risky asset at date 2 when the price sequence or a public disclosure can fully reveal private signals, we will not discuss trading volume and the properties of market liquidity about this case. Rather we analyze the case in which the price sequence cannot fully reveal private information. At date 2, the liquidity trader's expected profit is

According to eq. (8), substituting G into k, we can obtain the coefficient k is independent of  $\sigma_v^2$ . That is, the precision of the common error of the date 2 public signal,  $1/\sigma_v^2$ , is uncorrelated with  $\widetilde{x}_{i1}$ , and the date 1 decision is independent of the date 2 public signal.

$$E\left[\tilde{x}_{h2}\left(\tilde{F}-\tilde{P}_{2}\right)\right]$$

$$=E\left[\tilde{x}_{h2}\left\{\tilde{F}-b_{1}\left(\sum_{i=1}^{N}\tilde{x}_{i2}+\tilde{X}_{L2}\right)-b_{2}\tilde{Z}\right\}\right]$$

$$=-b_{1}E\left[\tilde{x}_{h2}\right]$$

$$=-b_{1}$$

$$(10)$$

We now examine how market liquidity is measured. According to Kyle (1985), we denote the market depth by  $1/b_1$  in eqs. (4) and (10).<sup>21</sup> The market depth,  $1/b_1$ , represents the order flow necessary to induce prices to rise or fall by one dollar. If  $b_1$  is small, a liquidity trader can buy or sell a large quantity of the risky asset at a price very close to the current market price. In this case, the market is viewed as a liquid market. In contrast, a large  $b_1$  implies that there is an illiquid market. If  $b_1 = 0$ , the market is infinitely deep. From eq. (10), a liquidity trader always chooses to trade at the date when  $b_1$  is small.

Furthermore, assume that the precision of all available public information, denoted by g, takes the form  $g = Var^{-1}(\widetilde{F}|\widetilde{Z}) = (\sigma_F^2 + \sigma_v^2)/\sigma_F^2\sigma_v^2$ . That is, the precision of a public disclosure, g, is decreasing in either the variance of the error in the disclosure,  $\sigma_v^2$ , or the variance of the terminal value of the risky asset,  $\sigma_F^2$ . We now turn to perform comparative statics over  $1/b_1$  for various exogenous parameters. The following proposition is obtained from  $\ell = L$  and  $b_1$  in eq. (4).

**Proposition 4.** At date 2, market liquidity is increasing in both the precision of a public disclosure and the number of the liquidity traders, and decreasing in the diversity of opinion among informed traders. That is, d(1/b)/dg > 0,  $d(1/b)/(d1-)\rho < 0$ , and d(1/b)/dL > 0. #

Proposition 4 is intuitive. It asserts that market liquidity increases as there is

 $<sup>^{21}\,</sup>$  Kyle (1985) measures the market depth by using the inverse of  $\,b_{\rm l}$  .

more public disclosures or as there is less diversity of opinion.  $^{22}$  In addition, market liquidity is higher if the number of liquidity traders trading (L) is large.  $^{23}$  The second result of Proposition 4 shows that a public disclosure reduces information asymmetry.  $^{24}$  In general, bid-ask spreads are used to measure the degree of liquidity of firms' securities. By disclosing to the market maker information known to the informed traders, the information asymmetry problem ameliorates and the transaction price quoted by the market maker is less sensitive to buy and sell orders. This implies that the market maker sets a low bid-ask spread when an announcement occurs that reduces information asymmetry and divergence of opinion. Therefore, the market becomes more liquid at the time of a public disclosure. The above discussion highlights the difference between our model and the analysis in Diamond and Verrecchia (1991) and Kim and Verrecchia (1994). We consider the market response when the sequence of prices fully or partially reveals private information. Hence, our model is more consistent with economic intuition than the previous articles.

Similarly, at date 1, a liquidity trader's expected profit is<sup>25</sup>

$$\begin{split} E\left[\tilde{x}_{h1}\left(\tilde{P}_{2}-\tilde{P}_{1}^{N}\right)\right] \\ = E\left[\tilde{x}_{h1}\left\{b_{1}\left(\sum_{i=1}^{N}\tilde{x}_{i2}+\tilde{X}_{L2}\right)+\right.\right. \left.\left.\left.b_{2}\tilde{Z}\right.\left(\sum_{i=1}^{N}+\tilde{x}_{i1}\right.\right.\left.\tilde{X}_{L1}^{N}\right\}\right] \end{split}$$

-

<sup>&</sup>lt;sup>22</sup> Because  $\rho$  is assumed to lie between (and including) 0 and 1, and N > 0, we obtain d(1/b)/dg > 0. Furthermore,  $db_1/d\rho < 0$  implies  $d(1/b)/(d1-)\rho < 0$ .

From eq. (4) and  $\ell = L$ , more liquidity traders (L) result into higher market liquidity  $(1/b_1)$ .

<sup>&</sup>lt;sup>24</sup> Kim and Verrecchia (1994) indicate that in the absence of announcements there are no opportunities for traders capable of informed judgments to exploit their ability to process information. This lessens the possibility of information asymmetries arising.

In eq. (7), the inverse of coefficient "a" can measure the market liquidity at date 1; hence, we denote the date 1 market depth by "1/a".

$$=b_{1}E\left[\widetilde{x}_{h2}^{2}\right]-aE\left[\widetilde{x}_{h1}^{2}\right]$$

$$=b_{1}-a$$
(11)

From eq. (11), if  $(b_1 - a)$  is large, a liquidity trader can earn more profit and a liquid market exists. In contrast, a small  $(b_1 - a)$  implies less profit earned by the liquidity trader and thus the market becomes illiquid. Because  $b_1 > 0$ , a small 'a' leads to larger  $(b_1 - a)$  and the liquidity traders have more profit, which results into a liquid market. On the contrary, a big 'a' leads to smaller  $(b_1 - a)$  and liquidity trading declines accordingly.<sup>26</sup>

Moreover, following (7) and  $Var^{-1}\left(\tilde{P}_2\left|\tilde{Z},\tilde{Y}_i\right)\right) = (\sigma_F^2 \left|\tilde{Q}_{DF}\right|^2 + \sigma_V^2 \left|\tilde{Z}_i\right|^2) / \left|\frac{2}{e}\right|^2 \left|\tilde{Z}_i\right|^2 + \left|\tilde{Z}_i\right|^2 +$ 

We now turn to trading volume and focus on the case of partially revealing. When the public disclosure is not a sufficient statistic for the private information, the date 2 trading volume, denoted by  $\tilde{V}_2$ , is simply

make less expected profit whereas the informed traders make more. Similarly, the precise public signals leads to less profit earned by the liquidity traders.

In our model, /a and  $1/b_1$  are used to measure the market liquidity of dates 1 and 2, respectively. Also note, a is influenced by N,  $\ell$ ,  $\sigma_F^2$ , and  $\sigma_e^2$ , while  $b_1$  is influenced by N,  $\ell$ ,  $\sigma_F^2$ ,  $\sigma_e^2$ , and  $\sigma_v^2$ . When the precision of the private signal is higher, the liquidity traders make less expected profit whereas the informed traders make more. Similarly, the precision of

$$\widetilde{V}_{2} = \frac{1}{2} \left\{ \sum_{i=1}^{N} |\widetilde{x}_{i2} - \widetilde{x}_{i1}| + \sum_{h=1}^{L} |\widetilde{x}_{h2} - \widetilde{x}_{h1}| \right\}$$
(12)

Trading volume is represented by one-half the absolute value of the change in the holdings by all traders. As described in eq. (12), the two components of  $\widetilde{V}_2$  represent the changes in the holdings of both informed and liquidity traders. Each component is the absolute value of a normally distributed random variable with zero mean, where  $\widetilde{x}_{i1}$  is independent of  $\widetilde{x}_{i2}$  and  $\widetilde{x}_{h1}$  is independent of  $\widetilde{x}_{h2}$ . As a result, expected trading volume at date 2, denoted by  $E(\widetilde{V}_2)$ , is

$$E(\widetilde{V}) = \sqrt{\frac{1}{2\pi}} \left\{ N \sqrt{Var(\widetilde{x}_{i2}) + Var(\widetilde{x}_{i1})} + L\sqrt{\frac{3}{4}} \right\}$$
(13)

Similarly, the date 1 trading volume, denoted by  $\widetilde{V}_1$ , is

$$\widetilde{V}_{1} = \frac{1}{2} \left\{ \sum_{i=1}^{N} \left| \widetilde{x}_{i1} \right| + \sum_{h=1}^{L} \left| \widetilde{x}_{h1} \right| \right\}$$

$$(14)$$

The expected trading volume at date 1, denoted by  $E(\widehat{V})$ , is

<sup>27</sup> Intuitively, the variance of holding units at date 2 is negatively related to the precision, represented by the inverse of variance, of the date 1 private signal and the date 2 public announcement, and the variance of demand for the risky asset at date 1 is negatively related to the precision of the date 1 private signal.

$$E(\widetilde{V}) = \sqrt{\frac{1}{2\pi}} \left\{ N \sqrt{Var(\widetilde{x}_i)} + \frac{1}{4} \right\}$$
 (15)

According to eq. (15), the amount of trading at date 1 depends not only on the volatility of informed trader i's date 1 holding units but also on the numbers of both liquidity and informed traders trading at that time. At date 1, trading volume is increasing in the variance of informed traders' risky asset holdings. And, when the numbers of both informed and liquidity traders are larger, more trading volume at date 1 would occur. By using the coefficients  $\theta_1$  and  $\theta_2$  in eq. (5), and coefficient a in eq. (7), we summarize the relevant results as follows.

#### Lemma 1.

- (1) If the sequence of prices partially reveals private information, trading volume of the risky asset arises from both informed and liquidity traders. However, if the sequence of prices fully reveals private information, all informed traders don't submit informed trading orders at date 2.
- (2) If the sequence of prices cannot fully reveal private information, the expected trading volume is higher at date 2 than at date 1 if and only if

$$k < \frac{N^{2} \left(\theta_{1}^{2} \Phi_{F}^{2} + \frac{\theta}{\ell}\right) \sigma \left(\frac{2}{r} + \frac{\theta}{\ell}\right) \sigma \left(\frac{2}{r} + \frac{\theta}{\ell}\right) \sigma \left(\frac{2}{r} + \frac{2}{r} + \frac{\theta}{\ell}\right) \sigma \left(\frac{2}{r} + \frac{2}{r} + \frac{2}{r}\right) - L^{2} \left(1 - \sqrt{2}\right)^{2}}{2NL(\sigma_{F}^{2} + \sigma_{g}^{2})^{2} + \left(1 - \sqrt{2}\right)}$$

The intuition is that a public disclosure improves the adverse selection problems. The market becomes more liquid at the time of a public disclosure. More liquidity implies that trading activity increases around a public announcement. In contrast, Kim and Verrecchia (1994) suggest that earnings announcements prompt the market maker to increase the bid-ask spread during the brief window (perhaps one or two days) surrounding their release. When the information processors are significantly active, more trading volume may also appear despite the reduction in

liquidity. This is because the processors of a public disclosure have temporary information advantage. Unfortunately, concerning this issue our model cannot obtain a clear conclusion about their relationship. The second part of Lemma 1 is a rough description.

## **Conclusions**

The analysis of capital markets generally depends on assumptions about the structure of market information and about how the traders process information. The various equilibrium paradigms used in the research on asset markets behavior differ in their assumptions with regard to the amount of information conveyed in price and the information sets exploited by traders for their portfolio decisions. First, a two-period noisy rational expectations model is analyzed in this paper. The traders can privately acquire a signal and a public report is released at the first and second dates, respectively. We consider two settings where the sequence of prices or a public disclosure can either fully reveal or partially reveal the private signals. This paper demonstrates the volume responses to a financial accounting disclosure at the time of an announcement under these frameworks. Second, we examine how the level of information asymmetry and the magnitude of liquidity affect the magnitude of trading volume reaction.

The conclusion of this paper is as follows. If private signals are not fully revealed by either the sequence of prices or a public disclosure, they are also informative in the second round, and the informed traders again condition their decisions on private information in period 2. Therefore, the risky asset's trading occurs in the second trading round in which trading volume arises from both informed and liquidity trading. However, if the private signal is fully revealed, no informed traders submit informed trading orders for the risky asset in the second

round, and thus all demand orders for the risky asset are from liquidity traders. Because a public accounting disclosure makes the transaction prices quoted by market makers less sensitive to buy and sell orders, it can mitigate the adverse selection problem. This implies that the market becomes more liquid at the time of a public announcement. Moreover, a less precise earnings announcement would lead to a greater diversity of opinions among individual informed traders. Less precise disclosures create information asymmetry between these informed traders and the market maker, and market becomes less liquid. Finally, it is intuitively true that an increase in the number of liquidity traders directly improves market liquidity.

Informed traders may receive private signals of different precision at costs that reflect the differential quality of information acquired privately. Because increases in the precision of disclosure can prompt differential private signal gathering prior to a disclosure, the problem of information asymmetry exacerbates at the time of a public disclosure. However, by providing a disclosure that dominates traders' private belief, information asymmetry decreases in the precision of a disclosure. Our model assumes that each informed trader receives private observation of identical precision although traders make different private acquisitions. Therefore, an interesting extension of our model is to allow for private signals of differential quality among traders. Furthermore, in our model, we assume that the number of informed traders is exogenous, but it may be determined endogenously. In this setting, researchers can further investigate how the number of informed traders is determined and what influence it has on the equilibrium model. Admati and Pfleiderer (1988) suggest that the discretionary liquidity traders have impetus to trade in periods of the greatest liquidity. It implies that the liquidity traders will not trade until the public information gets disseminated. This also provides a further discussion of our model developed above.

# **Appendix**

#### Appendix 1:

By using the standard formula the conditional expectation can be expressed as

$$\begin{split} &E\left(\widetilde{F}\middle|\widetilde{Y}_{i},\widetilde{Z}\right) \\ &= E\left(\widetilde{F}\right) + \beta_{1}\widetilde{Y}_{i} + \beta_{2}\widetilde{Z} \\ &= \left(\frac{\sigma_{F}^{2}\sigma_{v}^{2}}{\sigma_{F}^{2}\sigma_{v}^{2} + \sigma_{e}^{2}\sigma_{F}^{2} + \sigma_{e}^{2}\sigma_{v}^{2}}\right)\widetilde{Y}_{i} + \left(\frac{\sigma_{F}^{2}\sigma_{e}^{2}}{\sigma_{F}^{2}\sigma_{v}^{2} + \sigma_{e}^{2}\sigma_{v}^{2} + \sigma_{e}^{2}\sigma_{v}^{2}}\right)\widetilde{Z} \end{split} \tag{A.1}$$

$$&E\left(\widetilde{Y}_{j}\middle|\widetilde{Y}_{i},\widetilde{Z}\right) \\ &= E\left(\widetilde{Y}_{j}\right) + d_{1}\widetilde{Y}_{i} + d_{2}\widetilde{Z} \\ &= \left(\frac{\sigma_{v}^{2}\sigma_{F}^{2} + \rho_{e}^{2}\sigma_{F}^{2} + \rho_{e}^{2}\sigma_{F}^{2} + \rho_{e}^{2}\sigma_{v}^{2}}{\sigma_{F}^{2}\sigma_{v}^{2} + \sigma_{e}^{2}\sigma_{F}^{2} + \sigma_{e}^{2}\sigma_{v}^{2}}\right)\widetilde{Y}_{i} + \left(\frac{\sigma_{F}^{2}\sigma_{e}^{2} - \rho_{e}^{2}\sigma_{F}^{2}}{\sigma_{F}^{2}\sigma_{v}^{2} + \sigma_{e}^{2}\sigma_{F}^{2} + \sigma_{e}^{2}\sigma_{v}^{2}}\right)\widetilde{Z} \tag{A.2}$$

Therefore, the optimal  $\tilde{x}_{i2}$  is chosen to maximize

$$\begin{aligned} &\max_{\widetilde{X}_{i2}} E\left[\widetilde{X}_{i2}\left(\widetilde{F} - \widetilde{P}\right) \middle| \widetilde{Y}_{i}, \widetilde{Z}\right] \\ &= \max_{\widetilde{X}_{i2}} E\left[\widetilde{X}_{i2}\left\{\widetilde{F} - \left(b_{1}\widetilde{X}_{2} + b_{2}\widetilde{Z}\right) \middle| \widetilde{Y}_{i}, \widetilde{Z}\right]\right] \\ &= \max_{\widetilde{X}_{i2}} E\left[\widetilde{X}_{i2}\left\{\widetilde{F} - \left(b_{1}\left[\widetilde{X}_{I2} + \widetilde{X}_{L}\right] + b_{2}\widetilde{Z}\right) \middle| \widetilde{Y}_{i}, \widetilde{Z}\right]\right] \\ &= \max_{\widetilde{X}_{i2}} E\left[\widetilde{X}_{i2}\left\{\widetilde{F} - \left(b_{1}\left[\widetilde{X}_{i2} + \sum \widetilde{X}_{j2} + \widetilde{X}_{L}\right] + b_{2}\widetilde{Z}\right) \middle| \widetilde{Y}_{i}, \widetilde{Z}\right] \end{aligned}$$

$$= \frac{\max_{\widetilde{X}_{i,2}} E\left[\widetilde{X}_{i,2}\left\{\widetilde{F} - \left(b_1\left[\widetilde{X}_{i,2} + \theta_1\sum\widetilde{Y}_j + (N-1)\theta_2\widetilde{Z} + \widetilde{X}_{L_2}\right] + b_2\widetilde{Z}\right\}\right] \left|\widetilde{Y}_i,\widetilde{Z}\right|$$

By using eqs. (A.1) and (A.2), we get

$$\widetilde{x}_{i2} \left[ \frac{\sigma_F^2 \sigma_e^2 \widetilde{Z} + \sigma_F^2 \sigma_v^2 \widetilde{Y}_i}{\sigma_F^2 \sigma_v^2 + \sigma_e^2 \sigma_F^2 + \sigma_e^2 \sigma_v^2} - b_1 \widetilde{x}_{i2} - \left( b_2 + \theta_2 b_1 (N - 1) \right) \widetilde{Z} \right]$$

$$- \frac{(N - 1)b_1 \theta_1 \left( \sigma_v^2 \sigma_F^2 + \sigma_e^2 \sigma_F^2 + \sigma_v^2 \sigma_v^2 \right)}{\sigma_F^2 \sigma_v^2 + \sigma_e^2 \sigma_F^2 + \sigma_e^2 \sigma_v^2} \widetilde{Y}_i - \frac{(N - 1)b_1 \theta_1 \left( \sigma_F^2 \sigma_e^2 - \rho_e^2 \sigma_F^2 \right)}{\sigma_F^2 \sigma_v^2 + \sigma_e^2 \sigma_F^2 + \sigma_e^2 \sigma_v^2} \widetilde{Z} \right] (A.3)$$

Differentiating eq. (A.3) with respect to  $\tilde{x}_{i2}$  gives eq. (3).

#### Appendix 2:

In eq. (3), the coefficients of  $\widetilde{Y}_i$  and  $\widetilde{Z}$  are  $\theta_1$  and  $\theta_2$ , respectively.  $\theta_1$  and  $\theta_2$  can be simplified as

$$\theta_{1} = \frac{\sigma_{F}^{2} \sigma_{v}^{2} - (N-1)b_{1}\theta_{1} \left(\sigma_{F}^{2} \sigma_{v}^{2} + \sigma_{e}^{2} \sigma_{F}^{2} + \sigma_{v}^{2} \sigma_{e}^{2}\right)}{2b_{1} \left(\sigma_{F}^{2} \sigma_{v}^{2} + \sigma_{F}^{2} \sigma_{e}^{2} + \sigma_{e}^{2} \sigma_{v}^{2}\right)}$$

$$= \frac{\sigma_{F}^{2} \sigma_{v}^{2}}{2b_{1} \left(\sigma_{F}^{2} \sigma_{v}^{2} + \sigma_{F}^{2} \sigma_{e}^{2} + \sigma_{e}^{2} \sigma_{v}^{2}\right) + (N-1)b_{1} \sigma_{F}^{2} \sigma_{v}^{2} + \sigma_{e}^{2} \sigma_{F}^{2} + \sigma_{v}^{2} \sigma_{e}^{2}}$$

$$= \frac{\sigma_{F}^{2} \sigma_{v}^{2}}{b_{1} \left\{(N+1)\sigma_{F}^{2} \sigma_{v}^{2} + \left[2 + (N-1)\rho\right]\sigma_{e}^{2} \left(\sigma_{F}^{2} + \sigma_{v}^{2}\right)\right\}}$$
(A.4)

$$\theta_{2} = \frac{1}{2b_{1}} \left[ -(b_{2} + \theta_{2}b_{1}(N-1)) + \frac{\sigma_{F}^{2}\sigma_{e}^{2} - (N-1)b_{1}\theta_{1}(\sigma_{F}^{2}\sigma_{e}^{2} - \rho_{e}^{-2}\sigma_{F}^{2})}{\sigma_{F}^{2}\sigma_{v}^{2} + \sigma_{F}^{2}\sigma_{e}^{2} + \sigma_{e}^{2}\sigma_{v}^{2}} \right]$$

$$= \frac{1}{2b_{1}} \left[ -(b_{2} + \theta_{2}b_{1}(N-1)) + \frac{\sigma_{F}^{2}\sigma_{e}^{2} - (N-1)b_{1}\theta_{1}(\sigma_{F}^{2}\sigma_{e}^{2} - \rho_{e}^{-2}\sigma_{F}^{2})}{\sigma_{F}^{2}\sigma_{v}^{2} + \sigma_{F}^{2}\sigma_{e}^{2} + \sigma_{e}^{2}\sigma_{v}^{2}} \right]$$

$$+ \frac{\sigma_F^2 \sigma_e^2 \{ (N+1) \sigma_F^2 \sigma_v^2 + [2 + (N-1) \rho] \sigma_e^2 (\sigma_F^2 + \sigma_v^2) \} - (N-1) \sigma_F^2 (\sigma_v^2 \sigma_F^2 \sigma_e^2 - \rho_F^2) \sigma_e^2}{(\sigma_F^2 \sigma_v^2 + \sigma_F^2 \sigma_e^2 + \sigma_e^2 \sigma_v^2) \{ (N+1) \sigma_F^2 \sigma_v^2 + [2 + (N-1) \rho] \sigma_e^2 (\sigma_F^2 + \sigma_v^2) \}} \\
= \frac{1}{2b_1} \left[ -(b_2 + \theta_2 b_1 (N-1)) + \frac{\sigma_F^2 \sigma_e^2 \{ 2\sigma_F^2 \sigma_v^2 + (N-1) \rho_F^2 \sigma_v^2 + 2\sigma_e^2 \sigma_F^2 + 2\sigma_e^2 \sigma_v^2 + (N-1) \rho_F^2 \sigma_v^2 + \sigma_v^2 \}}{(\sigma_F^2 \sigma_v^2 + \sigma_F^2 \sigma_e^2 + \sigma_e^2 \sigma_v^2) \{ (N+1) \sigma_F^2 \sigma_v^2 + [2 + (N-1) \rho] \sigma_e^2 (\sigma_F^2 + \sigma_v^2) \}} \right] \\
= \frac{1}{(N+1)b_1} \left\{ -b_2 + \frac{\sigma_F^2 \sigma_e^2 [2 + (N-1) \rho]}{(N+1)\sigma_F^2 \sigma_v^2 + [2 + (N-1) \rho] \sigma_e^2 (\sigma_F^2 + \sigma_v^2)} \right\} \tag{A.5}$$

#### **Appendix 3:**

$$\tilde{P}_2 = E \left[ \tilde{F} \middle| \tilde{X}_2, \tilde{\bar{Z}} \right] = b_1 \tilde{X}_2 + b_2 \tilde{Z}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \frac{1}{\det} \begin{pmatrix} \sigma_F^2 + \sigma_v^2 & -N\theta_2 \left(\sigma_F^2 + \sigma_v^2\right) - N\theta_1 \sigma_F^2 \\ -N\theta_2 \left(\sigma_F^2 + \sigma_v^2\right) & N^2 \theta_2^2 \left(\sigma_F^2 + \sigma_v^2\right) + \ell + N\theta_1^2 \sigma_F^2 + \sigma_e^2 \\ -N\theta_1 \sigma_F^2 & +2\theta_1 \theta_2 N^2 \sigma_F^2 + \theta_1^2 N(N-1) \left(\sigma_F^2 + \sigma_e^2\right) \end{pmatrix} \begin{pmatrix} N\sigma_F^2 \left(\theta_1 + \theta_2\right) \\ \sigma_F^2 \end{pmatrix}$$

$$= \frac{1}{\det} \left( \frac{N\sigma_F^2 \sigma_v^2 \theta_1}{\sigma_F^2 \ell - N^2 \theta_1 \theta_2 \sigma_F^2 \sigma_v^2 + N\theta_1^2 \sigma_e^2 \sigma_F^2 + \theta_1^2 N(N-1) \left(\sigma_F^2 + \boldsymbol{\sigma}_e^2\right) \sigma_F^2} \right)$$

$$\det = \left\{ N^2 \theta_2^2 \left( \sigma_F^2 + \sigma_v^2 \right) + \ell + N \theta_1^2 \left( \sigma_F^2 + \sigma_e^2 \right) + 2 \theta_1 \theta_2 N^2 \sigma_F^2 + \theta_1^2 N (N - 1) \left( \sigma_F^2 + \sigma_e^2 \right) \right\} \left( \sigma_F^2 + \sigma_v^2 \right) - \left\{ N \theta_2 \left( \sigma_F^2 + \sigma_v^2 \right) + N \theta_1 \sigma_B^{\frac{1}{2}} \right\}^2$$

$$= \ell \left(\sigma_F^2 + \sigma_v^2\right) + \frac{N\sigma_F^4\sigma_v^4 \left\{N\sigma_F^2\sigma_v^2 + \left[1 + (N-1)\rho\right]\sigma_e^2 \left(\sigma_F^2 + \sigma_v^2\right\}\right\}}{b_1^2 \left\{(N+1)\sigma_F^2\sigma_v^2 + \left[2 + (N-1)\rho\right]\sigma_e^2 \left(\sigma_F^2 + \sigma_v^2\right\}\right\}^2}$$

Substituting  $\theta_1$  and "det" into  $b_1$ , the collecting terms provide:

$$b_{1} = \frac{Nor_{r}^{4} \ ^{4}b_{1} \Big\{ (N + \sigma tb) \ ^{2}_{F} \ ^{2}_{v} + \frac{1}{2} - (\partial t \sigma 1)d_{1}^{2} \ ^{2}_{F} \Big( \ ^{2}_{F} \ \ ^{2}_{v} \Big) \Big\}}{\ell \Big( \sigma_{F}^{2} + \sigma \ ^{2}_{v} \Big) b_{1}^{2} \Big\{ (N + \sigma tb) \ ^{2}_{F} + ^{2}_{v} + \Big[ 2 - (\partial t \sigma 1)d_{1}^{2} + ^{2}_{e} \Big( \ ^{2}_{F} \sigma \sigma \ ^{2}_{v} \Big) \sigma^{2}_{v} \ N \ ^{4}_{F} \ ^{4}_{v} \Big\{ N \sigma^{2}_{F} \sigma^{2}_{v} \sigma \left[ 1 \ (N \ 1) \ \right] \ ^{2}_{e} \Big( \ ^{2}_{F} \ \ ^{2}_{v} \Big) \Big\}}$$

$$= \sqrt{\frac{N\sigma_F^4 \sigma_v^4 \left(\sigma_F^2 \sigma_v^2 + \sigma_F^2 \sigma_e^2 + \sigma_e^2 \sigma_v^2\right)}{\ell \left(\sigma_F^2 + \sigma_v^2\right)^2 \left\{\left(N + \right)\sigma_F^2 \sigma_v^2 + \left[2 + (N - 1)\rho\right]\sigma_e^2 \left(\sigma_F^2 + \sigma_v^2\right)^2}}$$
(A.6)

$$b_{2} = \frac{1}{\det} \left\{ \sigma_{F}^{2} \ell + \frac{N \sigma_{F}^{4} \sigma_{v}^{4} \left[ \sigma_{e}^{2} \sigma_{F}^{2} + (N-1) \left( \sigma_{F}^{2} + \rho_{e}^{-2} \right) \sigma_{F}^{2} \right]}{b_{1}^{2} \left\{ (N+1) \sigma_{F}^{2} \sigma_{v}^{2} + \left[ 2 + (N-1) \rho \right] \sigma_{e}^{2} \left( \sigma_{F}^{2} + \sigma_{v}^{2} \right) \right\}^{-2}} - \frac{N \sigma_{F}^{4} \sigma_{v}^{4} \left[ -b_{2} \left\{ (N+1) \sigma_{F}^{2} \sigma_{v}^{2} + \left[ 2 + (N-1) \rho \right] \sigma_{e}^{2} \left( \sigma_{F}^{2} + \sigma_{v}^{2} \right) \right\} + \sigma_{F}^{2} \sigma_{e}^{2} \left\{ 2 + (N-1) \rho \right\}}{b_{1}^{2} \left\{ (N+1) \sigma_{F}^{2} \sigma_{v}^{2} + \left[ 2 + (N-1) \rho \right] \sigma_{e}^{2} \left( \sigma_{F}^{2} + \sigma_{v}^{2} \right) \right\}^{-2} (N+1)} \right\}$$

$$\equiv \frac{\mathbf{A}}{\mathbf{R}}$$

where

$$\mathbf{A} = \sigma_F^2 \left\{ \ell + \frac{N\sigma_F^2 \sigma_V^4 \left( \frac{2}{e} + \frac{2}{F} + (N\sigma_F - 1) \left( \frac{e^2}{e^2} + \frac{2}{e} + \frac{2}{F} \right) \right)}{b_1^2 \left\{ (N+1)\sigma_F^2 \sigma_V^2 + \left[ \frac{12}{F} + (N\sigma_F - 1) \left( \frac{e^2}{F} + \frac{2}{e^2} + \frac{2}{F} \right) \right\}^2 - \frac{N^2 \sigma_F^2 \sigma_V^4 \left( \frac{2}{F} + \frac{2}{e^2} \left[ 2 + (N\sigma_F - 1) \right] \right)}{b_1^2 \left\{ (N+1)\sigma_F^2 \sigma_V^2 + \left[ \frac{12}{F} + (N\sigma_F - 1) \left( \frac{2}{F} + \frac{2}{F} \right) \right] \right\}^2 (N+1)} \right\}$$

$$\begin{aligned} \mathbf{B} &= \left(\sigma_F^2 + \sigma_v^2\right) \left\{ e^{\frac{2}{h}} \frac{N\sigma_F^2 \sigma_v^4 \sigma_v^4 \sigma_v^2 - \frac{2}{e} + (N\sigma_v^2 \sigma_v^4) - \frac{2}{e} + \frac{2}{F}}{b_1^2 \left\{ (N+1)\sigma_F^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{e} + \frac{2}{F} \right] \right\}^2} - \frac{N^2 \sigma_F^2 \sigma_v^4 \sigma_v^4 \sigma_v^4 - \frac{2}{e} \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{e} + \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_F^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{e} + \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_F^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{e} + \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_F^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{e} + \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_F^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{e} + \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_F^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_F^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_F^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_F^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_V^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_V^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_v^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_v^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_v^2 \sigma_v^2 + \left[ 2 + (N\sigma_v^2 \sigma_v^4) - \frac{2}{F} \right] - \frac{2}{h_1^2 \left\{ (N+1)\sigma_v^2 - \frac{2}{F} \right\} - \frac{2}{h_1$$

Substituting  $b_1$  into **A** and **B**, the collecting terms provide:

$$b_{2} = \frac{\sigma_{F}^{2} \left\{ 1 + \frac{\left(\sigma_{F}^{2} + \sigma_{v}^{2}\right) \left[\sigma_{e}^{2}\sigma_{F}^{2} + (N - (1)\sigma_{F}^{2} + \rho)_{e}^{2}\sigma_{F}^{2}\right] - \left(\sigma_{F}^{2} + \sigma_{v}^{2}\right) N \left\{\sigma_{F}^{2}\sigma_{e}^{2} \left[2 + (N - 1)\rho\right]\right\}}{\sigma_{F}^{2} \left(\sigma_{F}^{2}\sigma_{v}^{2} + \sigma_{F}^{2}\sigma_{e}^{2} + \sigma_{e}^{2}\sigma_{v}^{2}\right)} \right\}}$$

$$\frac{\left(\sigma_{F}^{2} + \sigma_{v}^{2}\right) \left\{ 1 + \frac{\left(\sigma_{F}^{2} + \sigma_{v}^{2}\right) \left[\sigma_{e}^{2}\sigma_{F}^{2} + (N - 1)\rho_{e}^{2}\sigma_{F}^{2}\right] - \left(\sigma_{F}^{2} + \sigma_{v}^{2}\right) N \left\{\sigma_{F}^{2}\sigma_{e}^{2} \left[2 + (N - 1)\rho\right]\right\}}{\sigma_{F}^{2} \left(\sigma_{F}^{2}\sigma_{v}^{2} + \sigma_{F}^{2}\sigma_{e}^{2} + \sigma_{e}^{2}\sigma_{v}^{2}\right)} \right\}}$$

$$\equiv \frac{\mathbf{C}}{} \tag{A.7}$$

where

#### **Appendix 4:**

Substituting (A.6) into (A.4) provides

$$\theta_{1} = \frac{\sigma_{F}^{2} \sigma_{v}^{2}}{\sigma_{F}^{2} \sigma_{v}^{2} \sqrt{\frac{N(\sigma_{F}^{2} \sigma_{v}^{2} + \sigma_{F}^{2} \sigma_{e}^{2} + \sigma_{e}^{2} \sigma_{v}^{2})}{\ell(\sigma_{F}^{2} + \sigma_{v}^{2})}}}$$

$$= \sqrt{\frac{\ell(\sigma^{2} + \sigma_{v}^{2})}{N(\sigma^{2} \sigma_{v}^{2} + \sigma^{2} \sigma^{2} + \sigma^{2} \sigma_{v}^{2})}}$$
(A.8)

Similarly, substituting (A.7) into (A.5) yields

$$\theta_{2} = \frac{-1}{(N+1)} \sqrt{\frac{\ell(\sigma_{F}^{2} + \sigma^{2}) \left\{ (N + \sigma \sigma^{2}_{F} + \ell^{2} + \ell - \ell N \sigma d - \sigma \ell^{2} - \ell^{2}_{F} - \ell^{2}_{F}) \right\}}{N \sigma \sigma \sigma \sigma^{2}_{F} \sigma \sigma^{2}_{F} + \sigma^{2}_{F} - \ell^{2}_{F} - \ell^{2}_{F}}} \bullet \frac{\mathbf{C}}{\mathbf{D}}$$

$$+ \frac{\sigma^{2} \left[ + (N - 1 \rho) \right]}{\sigma^{2} (N+1)} \sqrt{\frac{\ell(\sigma_{F}^{2} + \sigma^{2}_{F})}{N(\sigma \sigma^{2}_{F} - \sigma \sigma \sigma^{2}_{F} - \sigma \sigma^{2}_{F} - \ell^{2}_{F}})}}$$

$$\equiv \frac{\mathbf{E}}{\mathbf{E}} \tag{A.9}$$

#### **Appendix 5:**

$$\begin{split} E\left[\tilde{x}_{i1}\left(\tilde{P}_{2}-\tilde{P}_{i}\right)\left|\tilde{Y}_{i}\right] \\ &=E\left[\tilde{x}_{i1}\left\{\left(b_{1}\tilde{X}_{2}+b_{2}\tilde{Z}-a\tilde{X}\right)\left|\tilde{Y}_{i}\right|\right.\right] \\ &=\tilde{x}_{i1}\left\{b_{1}\theta_{1}\tilde{Y}_{i}+B_{1}\left(N-1\right)E\left[\tilde{Y}_{j}\left|\tilde{X}_{i}\right|\right]\right. \quad b_{i}^{2}N_{2}\left[E\left[\tilde{Z}\right]\tilde{Y}_{i}\right] \quad \left[b_{2}E\left[\tilde{Z}\right]\tilde{Y}_{i}-a\tilde{x}_{i1}-ak(N-1)E\right]\tilde{Y}_{j}\left|\tilde{Y}_{i}\right\} \end{split} \tag{A.10}$$

Differentiating eq. (A.10) with respect to  $\tilde{x}_{i1}$  yields

$$\widetilde{x}_{i1} = \frac{1}{2a} \left\{ b_1 \theta_1 + (b_1 \theta_1 - ak)(N - 1) \frac{\sigma_F^2 + \rho_e^2}{\sigma_F^2 + \sigma_e^2} + (b_1 N \theta_2 + b_2) \frac{\sigma_F^2}{\sigma_F^2 + \sigma_e^2} \right\} \widetilde{Y}_i$$

$$\equiv k \widetilde{Y}_i \qquad (A.11)$$

Also, the date 1 equilibrium price for the risky asset can be written as

$$\widetilde{P}_{1} = E\left(\widetilde{F}\middle|\widetilde{X}_{1}\right)$$

$$= \frac{Nk\sigma_{F}^{2}}{\ell + Nk^{2}\left(\sigma_{F}^{2} + \sigma_{e}^{2}\right) + k^{2}N(N - (1)\sigma_{F}^{2} + \boldsymbol{\rho})_{e}^{2}}\widetilde{X}_{1}$$

$$\equiv a\widetilde{X}_{1} \tag{A.12}$$

#### Appendix 6:

Since 
$$\widetilde{x}_{i1} = k\widetilde{Y}_{i}$$

$$\widetilde{\gamma}_{2} = \beta \widetilde{Z} = \left(\widetilde{\gamma} \middle| \widetilde{Z} \right) = \frac{\sigma^{2}}{\sigma^{2} + \sigma_{v}^{2} + \sigma_{e}^{2}} \widetilde{Z}$$

$$\widetilde{\gamma}_{1} = \widetilde{X}_{1} = \left(\widetilde{\gamma} \middle| \widetilde{X} \right) = \frac{N \sigma^{2}}{\ell + N^{2} \left(\sigma^{2} + \sigma_{e}^{2}\right) + \frac{2}{N} N - \left(\sigma^{2} + \sigma\right)_{e}^{2}} \widetilde{X}_{1}$$

By using above relations, differentiating  $E\left[\tilde{x}_{i1}\left(\tilde{P}_{2}-\tilde{P}_{j}\right)\middle|\tilde{Y}_{i}\right]$  with respect to  $\tilde{x}_{i1}$  provides

$$\widetilde{x}_{i1} = \frac{1}{2a} \left\{ \beta - ak(N-1) \left( \frac{\sigma_F^2 + \rho_e^2}{\sigma_F^2 + \sigma_e^2} \right) \right\} \widetilde{Y}_i \equiv k \widetilde{Y}_i$$
(A.13)

Substituting  $\beta$  and a into k provides

$$k = \sqrt{\frac{\sigma_F^2 \ell}{G(\sigma_F^2 + \sigma_v^2 + \sigma_\theta^2)}}$$
 (A.14)

where

$$G = N\sigma_F^2 (N-1) \left( \frac{\sigma_F^2 + \sigma_e^2}{\sigma_F^2 + \sigma_e^2} \right) - N\sigma_F^2 + \sigma_e^2 - N(N\sigma_F^2) + N(N\sigma_F^2) + \rho \sigma_e^2$$

## References

- Admati, A., and P. Pfleiderer. A theory of intraday patterns: volume and price variability. <u>The Review of Financial Studies</u>. 1. 1988: 3-40.
- Atiase, R., and L. Bamber. Trading volume reactions to annual accounting earnings announcements. <u>Journal of Accounting and Economics</u>. 17. 1994: 309-329.
- Bamber, L. Unexpected earnings, firm size, and trading volume around quarterly earnings announcements. The Accounting Review. 42. 1987: 510-532.
- Brown, D. P., and R. H. Jennings. On technical analysis. <u>The Review of Financial Studies</u>. 2. 1989: 527-551.
- Bushman, R. Public disclosure and the structure of private information markets. Journal of Accounting Research. 29. 1991: 261-276.
- Demski, J., and G. Feltham. Market response to financial reports. <u>Journal of Accounting and Economics</u>. 17. 1994: 3-40.
- Diamond, D. Optimal release of information by firms. <u>Journal of Finance</u>. 40. 1985: 1071-1094.
- Diamond, D., and R. Verrecchia. Disclosure, liquidity, and the cost of capital. <u>Journal of Finance</u>. 46. 1991: 1325-1359.
- Grundy, B., and M. McNichols. Trade and revelation of information through prices and direct disclosure. <u>The Review of Financial Studies</u>. 3. 1989: 495-526.
- Hakansson, N., J. Kunkel, and J. Ohlson. Sufficient and necessary conditions for information to have value in pure exchange. <u>Journal of Finance</u>. 37. 1982: 1169-1181.
- Karpoff, J. A theory of trading volume. <u>Journal of Finance</u>. 41. 1986: 1069-1087.

- Karpoff, J. The relation between price changes and trading volume: a survey. Journal of Financial and Quantitative Analysis. 22. 1987: 109-126.
- Kim, O., and R. Verrecchia. Trading volume and price reactions to public announcements. <u>Journal of Accounting Research</u>. 29. 1991a: 302-321.
- Kim, O., and R. Verrecchia. Market reaction to anticipated announcements. Journal of Financial Economics. 30. 1991b: 273-309.
- Kim, O., and R. Verrecchia. Market liquidity and volume around earnings announcements. Journal of Accounting and Economics. 17. 1994: 42-67.
- Kyle, A. Continuous auctions and insider trading. <u>Econometrica</u>. 53. 1985: 1315-1335.
- Lin, Y., and T. Wang. The effect of sequential information releases on trading volume and price behavior. <u>Accounting and Business Research</u>. 31. 2001: 119-132.
- Lin, Y., T. Wang, and Y. Tsai. Responses of trading volume and price on private information. The Chinese Accounting Review. 29. 1995: 1-40.
- Lundholm, R. Public signals and the equilibrium allocation of information. <u>Journal of Accounting Research</u>. 29. 1991: 322-349.
- Lys, T., and S. Sohn. The association between revisions of financial analysts' earnings forecasts and security-price changes. <u>Journal of Accounting and Economics</u>. 13. 1990: 341-363.
- McNichols, M., and B. Trueman. 1994. Public disclosure, private information collection, and short-term trading. <u>Journal of Accounting and Economics</u>. 17. 1994: 69-94.

# 會計揭露對市場流動性及交易之影響

林宜勉\*

# 摘 要

本研究之目的擬先構建兩期干擾理性預期均衡模式,其中市場存在消息靈通交易者、流動性交易者與市場制定者等三種風險中立的參與者,而消息靈通交易者在第一期取得私有資訊並且公司在第二期揭露財務報表。文中將分別探討在序列價格可以完全顯露與部份顯露私有訊號的兩種架構下,有關公開揭露時的價格與交易量模式,並且分析會計資訊的揭露對資訊不對稱與市場流動性大小之影響。模式結果指出,若序列價格無法完全顯露私有資訊,市場在第二期盈餘宣告時消息靈通交易者對風險性資產將有交易產生;但當序列價格可以完全顯露私有資訊時,消息靈通交易者在第二期將不會交易風險性資產。同時,在前者架構下第二期的市場流動性為公開訊號精確度與流動性交易者人數的遞增函數、而為消息靈通交易者意見分歧的遞減函數。

關鍵詞:公開揭露,資訊不對稱,市場流動性,干擾理性預期模式

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