# 製程能力指標 $C_{pm}$ 及 $C_{pp}$ 之應用

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## 摘 要

製程能力指標是製程參數(製程期望值、標準差)及規格之一項沒有單位的函數，它可說是衡量製程是否合乎品質水準的一種簡單易懂的語言；它經常被用來判定產品在特定公差下是否合乎品質水準(Boyles, 1991)。然而，不少人僅透過樣本算出製程能力指標值就來判定製程能力，Montgomery (1985)等不少學者即指出如此進行判定製程是不適當的。因此，本文係針對 $C_{pm}$ 指標的應用，於單製程時建構一 $C_{pm}$ 的推薦最小值的判定程序；及於多重製程時提出以 $C_{pp}$ 的多重製程績效分析圖的圖解法來分別判定多重製程是否合乎品質能力。透過本文所提之方法確實可以簡易且正確地判定出製程之品質能力。最後，透過半導體製程的兩個實際例子來說明 $C_{pm}$ 判定程序及 $C_{pp}$ 圖解法之應用。

**關鍵詞**：製程能力指標，推薦最小值，多重製程績效分析圖，$C_{pm}$ 及 $C_{pp}$ 指標

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# Certain Implementation of the PCIs $C_{pm}$ and $C_{pp}$

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# Abstract

A process capability index (PCI) is a unitless function of the process parameters ($\mu$, $\sigma$) and the process specifications (LSL, T, USL) designed to provide a common, easily understood language for quantifying the performance of a process (Boyles, 1991). It is used to determine whether a production process is capable of producing items within a specified tolerance. However, most people using the PCI simply consider the value of the index calculated from the given sample to make a conclusion on whether the given process is capable or not. Several authors (Montgomery (1985), Chou et al (1990) and Cheng (1992)) have pointed out that it is not an appropriate approach. Based on the application of Cpm index, this paper presents approaches for evaluating the quality capability of both the single process and multiple process in order to correctly and easily evaluate the quality performance of the process. Hence, this study constructs a testing procedure for RMV of Cpm to help judge whether a single process is capable or not, and proposes a graphic approach which uses the Cpp index for evaluating the quality performance of multiple processes. Finally, two numerical examples which illustrates the usage of the indices are provided.

**Keywords：** Process Capability Index, Recommended Minimum Value, Multiprocess Performance Analysis Chart, $C_{pm}$ and $C_{pp}$.

# INTRODUCTION

A manufacturing process is characterized by numerical measurements taken for certain quality characteristics of process operations or units produced by the process. A process specification generally consists of lower and upper specifications (LSL, USL) and a target value T is located somewhere between these limits. The process capability indices are appropriate measures of progress for quality improvement, in which reduction of variability is the guiding principle and process yield is the primary measure of success.

A process capability index is used to determine whether a production process is capable of producing items within a specified tolerance. The most widely used measures of process capability are $C_p$, $C_{pk}$ and $C_{pm}$, defined as

$$C_p = \frac{USL - LSL}{6\sigma} \quad \text{(Kane (1986))} \tag{1}$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu\text{-}LSL}{3\sigma}\right\} \tag{2}$$

and

$$C_{pm} = \frac{USL - LSL}{6\sqrt{E(X-T)^2}} = \frac{USL - LSL}{6\tau} \tag{3}$$

where $\tau = \sqrt{E(X-T)^2} = \sqrt{\sigma^2 + (\mu - T)^2}$ (Chan et al(1988))

Since the indices depend on the unknown parameters $\sigma$ and $\mu$, the true values of these indices may only be estimated. Estimators generally use $\overline{X}$ for $\mu$, S for $\sigma$, where $\overline{X}$ is the sample mean and S is the sample standard deviation, based on a random sample of n observations. However, many people merely look at the value of the index calculated from the given sample and then make a conclusion on whether the given process is capable or not, which is not an effective approach. To keep the capability of the process from being misjudged, Montgomery (1985) and Chou et al (1990) each utilized the RMV of both $C_p$ and $C_{pk}$ for the judgment function. However, no literature has ever applied the RMV of $\widehat{C}_{pm}$ in judging the capability of a process. Since, regarding the selection of an index, Boyles (1991) made a very thorough comparison and concluded that $C_{pm}$ is the most desirable index. Therefore, the first objective of this paper is to propose a testing procedure for RMV of $\widehat{C}_{pm}$ and then to evaluate the quality capability of the process.

Singhal (1991) has proposed the multiprocess performance analysis chart (MPPAC) to evaluate the process capabilities of many products simultaneously. The MPPAC is also based on PCI to analyze and evaluate the process capability of multiple processes by integrating the departure of process mean from target value, the process variability and the expected degree exceeding specification limits.

Greenwich and Jahr-Schaffrath (1995) introduced a new index, $C_{pp}$, which was a transformation from the $C_{pm}$ index ($C_{pp} = (\frac{1}{C_{pm}})^2$) .The $C_{pp}$ index could divide the reasons which caused process variance into departure from process mean and process variability, and this index provides uncontaminated separation between information concerning the process accuracy and process precession.

Combining the papers of Singhal (1990, 1991) and Greenwich et al (1995), author will develop a MPPAC of $C_{pp}$ and thus the process capability of multiple product processes can be easily analyzed.

To sum up, this paper establishes a testing procedure for the RMV of $\widehat{C}_{pm}$ to evaluate the single process capability. In addition, the MPPAC of $C_{pp}$ is developed to simultaneously evaluate the process capabilities of multiple product processes.

# TESTING PROCEDURE FOR THE RMV OF $\widehat{C}_{pm}$

Cheng [1992] presents a few tables and graphs which can be used to find the rejection probability, and to judge whether the process is capable or not through comparison with the rejection probability and the given allowable rejection probability. From a practical view, we consider that Cheng's approach is not convenient, because : (1) this approach must offer many different rejection probability tables depending on P values and $\tilde{C}_{pm}$ values, (2) in order to use the published tables, the sampling number must be 10, 15, ⋯, 50 and the process capability value C must be 1 or 1.33, (3) the process mean must be equal to the target value. In order to improve the Cheng's evaluation approach, the approach of the RMV of $\widehat{C}_{pm}$ is proposed. In this section, the RMV of $\widehat{C}_{pm}$ is derived, and a testing procedure is offered to judge whether a single quality characteristic is capable or not.

## A. Derivation of RMV of $\widehat{C}_{pm}$

Let $x_1, x_2, \cdots, x_n$ denote a random sample of n measurements on the process characteristic of interest. Let $\mu$ and $\sigma$ be the mean and standard deviation of the normal distribution of the process output. As both specification limits (LSL, USL) and the target value T are given thus there is a need to take into account departures from the target values, and a convenient way to measure process capability is to use $C_{pm}$.

In order to conveniently derive the recommended minimum value of $\widehat{C_{pm}}$, both the noncentral chi-square distribution should be defined and a theory should be proved as follow.

**Definition 1 :** $x_1, x_2, \cdots, x_n$ are independently distributed and $x_i$ is $N(\mu, \sigma)$, then $\frac{x_i - T}{\sigma} \sim N(\xi_i, 1)$ and then the random variable $U = \sum_{i=1}^{n} x_i^2$ is called a noncentral chi-square variable with n degrees of freedom. We call $\delta = (\sum_{i=1}^{n} \frac{x_i - T}{\sigma})^{\frac{1}{2}}$ the noncentrality parameter of the distribution.

**Theorem 1 :** The quantity $\frac{\widehat{\tau^2}}{\tau^2}$ is equal to $\frac{1}{n(1+\xi^2)} \chi^2_{n,\sqrt{n}|\xi|}$, where $\chi^2_{n,\sqrt{n}|\xi|}$ is a noncentral chi-square distribution with n degrees of freedom and noncentrality parameter $\sqrt{n}|\xi|$, and $\widehat{\tau^2} = \frac{1}{n}\sum_{i=1}^{n}(x_i - T)^2 = \widehat{\sigma^2} + (\bar{x} - T)^2$.

Proof of Theorem 1 :

Since $x_i \sim N(\mu, \sigma)$

$$x_i - T \sim N(\mu - T, \sigma)$$

$$E(\frac{x_i - T}{\sigma}) = \frac{E(x_i) - T}{\sigma} = \frac{\mu - T}{\sigma}, \text{ and let } \xi = \frac{\mu - T}{\sigma}$$

$$Var(\frac{x_i - T}{\sigma}) = \frac{1}{\sigma^2} Var(x_i) = \frac{\sigma^2}{\sigma^2} = 1$$

So, $\frac{x_i - T}{\sigma} \sim N(\xi_i, 1)$

we get $\sum_{i=1}^{n}(\frac{x_i - T}{\sigma})^2 \sim \chi^2_{n,\sqrt{n}|\xi|}$ (by the definition 1)

$$\sum_{i=1}^{n}(\frac{x_i - T}{\sigma})^2 = \frac{n \times \frac{1}{n}\sum_{i=1}^{n}(x_i - T)^2}{\sigma^2} = \frac{n\widehat{\tau^2}}{\sigma^2} = \frac{n(1+\xi^2)\widehat{\tau^2}}{(1+\xi^2)\sigma^2} \sim \chi^2_{n,\sqrt{n}|\xi|}$$

Since $\tau^2 = \sigma^2 + (\mu - T)^2 = \sigma^2(1 + (\frac{\mu - T}{\sigma})^2) = \sigma^2(1 + \xi^2)$

So $\frac{n(1+\xi^2)\widehat{\tau^2}}{(1+\xi^2)\sigma^2}=\frac{n(1+\xi^2)\widehat{\tau^2}}{\tau^2}\sim\chi'^2_{n,\sqrt{n}|\xi|}$.

Hence, $\frac{\widehat{\tau^2}}{\tau^2}\sim\frac{1}{n(1+\xi^2)}\chi'^2_{n,\sqrt{n}\xi}$.

Let $c = f(x_1, x_2, \cdots, x_n)$ be a statistic satisfying $p_r\{C_{pm} \geqq c\} = 1 - \alpha$, where $\alpha$ is significance level. Then c is a 100 (1 - $\alpha$)% lower confidence limit for $C_{pm}$. It follows that :

$$p_r\left\{C_{pm}\geq c\right\}=p_r\left\{\frac{C_{pm}}{\widehat{C}_{pm}}\geq\frac{c}{\widehat{C}_{pm}}\right\}$$

$$=p_r\left\{\frac{USL-LSL}{6\tau}\Big/\frac{USL-LSL}{\widehat{6\tau}}\geq\frac{c}{\widehat{C}_{pm}}\right\} \tag{4}$$

$$=p_r\left\{\frac{\widehat{\tau}}{\tau}\geq\frac{c}{\widehat{C}_{pm}}\right\}$$

$$=p_r\left\{\sqrt{\frac{1}{n(1+\xi^2)}\chi'^2_{n,\sqrt{n}|\xi|}}\geq\frac{c}{\widehat{C}_{pm}}\right\} \quad \text{(by Thm. 1)}$$

$$=p_r\left\{\widehat{C}_{pm}\cdot\sqrt{\frac{1}{n(1+\xi^2)}\chi'^2_{n,\sqrt{n}|\xi|}}\geq c\right\} \tag{5}$$

It is shown in Appendix I that equation (5) can be approximated by

$$p_r\left\{\left[\frac{1+2\xi^2}{n(1+\xi^2)^2}\cdot\chi^2_k\right]^{\frac{1}{2}}\cdot\widehat{C}_{pm}\geq c\right\}=1-\alpha \tag{6}$$

where $\chi^2_k$ is a Chi-square distribution with k degrees of freedom, $\alpha$ is the confidence level, and $k=\frac{n(1+\xi^2)^2}{(1+2\xi^2)}$. Therefore, the 100(1-$\alpha$)% lower confidence bound for $C_{pm}$ can be written in terms of $\widehat{C}_{pm}$ as

$$C = \widehat{C_{pm}} \cdot \left[ \frac{(1+2\xi^2)}{n(1+\xi^2)^2} \cdot \chi_k^2 \right]^{\frac{1}{2}} \tag{7}$$

on the other hand, $\widehat{C_{pm}}$ can be written as

$$\widehat{C_{pm}} = C \cdot \left[ \frac{(1+2\xi^2)}{n(1+\xi^2)^2} \cdot \chi_k^2 \right]^{-\frac{1}{2}} \tag{8}$$

and replacing the unknown parameter k by $\widehat{k}$, where $\widehat{k} = \frac{n(1+\widehat{\xi^2})^2}{(1+2\widehat{\xi^2})}$, $\widehat{\xi} = \frac{\overline{X}-T}{\widehat{\sigma}}$, $\widehat{\sigma} = \frac{(n-1)S}{n}$.

The current practice is to compare the estimated index value with a recommended minimum value. If the estimated index value is larger than or equal to the RMV, then the process is considered to be capable. However, the RMV is for the true index, not for the estimated index. Therefore, even if the estimated index is larger than or equal to the RMV for the true index, one can not be 100 % sure that the true index is larger than or equal to the RMV and 100 % confident in claiming that the process is capable. A process is called capable, similar to Juran's (1980) definition, if the $\widehat{C_{pm}}$ exceeds its limit. In practice, since $\widehat{C_{pm}}$ is unknown, we take a random sample of size n calculate $\widehat{C_{pm}}$, and use the RMV approach to judge the process. That is, if $\widehat{C_{pm}} \geqq C_0 \cdot \left[ \frac{(1+2\widehat{\xi^2})}{n(1+\widehat{\xi^2})^2} \cdot \chi_k^2 \right]^{-\frac{1}{2}}$ (where $C_0$ is the required process capable value), then we can claim that the process is capable at least 100 (1-$\alpha$)% of the time .

Therefore, the factor $C_0 \cdot \left[ \frac{(1+2\widehat{\xi^2})}{n(1+\widehat{\xi^2})^2} \cdot \chi_k^2 \right]^{-\frac{1}{2}}$ is the recommended minimum value of the estimated index $\widehat{C_{pm}}$ for the process to be considered capable at least 100(1-$\alpha$)% of the time.

## B. Testing Procedure

The process capability index is usually used to judge whether a manufacturing process is capable or not. We use the estimator $\widehat{C_{pm}}$, as the test statistic and will evaluate the recommended minimum value to make a decision. Testing the hypothesis should determine

$$\begin{cases} H_0 : \text{If } \widehat{C_{pm}} < C^* \text{ then the process is not capable;} \\ H_1 : \text{If } \widehat{C_{pm}} \geqq C^* \text{ then the process is capable, where } C^* = C_0 \cdot \left[ \dfrac{(1+2\widehat{\xi^2})}{n(1+\widehat{\xi^2})^2} \cdot \chi_k^2 \right]^{-\frac{1}{2}}. \end{cases}$$

Therefore, we will introduce a testing procedure which may guide the quality evaluator to judge whether the process is capable or not.

Step 1 : In a practical application, the practitioner should decide upon the following five values.

a. specification limits : LSL, USL

b. target value : T

c. the required process capability value : $C_0$

d. confidence level : $1-\alpha$

Step 2 : Take a random sample of size n and calculate $\widehat{C_{pm}} = \dfrac{USL - LSL}{6\sqrt{s^2 + (\overline{X} - T)^2}}$ (Boyles, 991).

Step 3 : Calculate the recommended minimum value $C^*$ by

$$C^* = C_0 \cdot \left[ \frac{(1+2\widehat{\xi^2})}{n(1+\widehat{\xi^2})^2} \cdot \chi_k^2 \right]^{-\frac{1}{2}}, \text{ where } C^* \text{ is the RMV of } \widehat{C_{pm}}.$$

Step 4 : Make a decision :

If $\widehat{C_{pm}} < C^*$, then we conclude that process is incapable,

If $\widehat{C_{pm}} \geqq C^*$, then we conclude that process is capable.

A SAS (1990) language computer program has been developed to ease calculations and to make a correct decision for users, who may just enter the required data.

The following example is used to illustrate the testing procedure of RMV developed in this paper better than Cheng's graphic method.

A customer provided the specification limits of inner diameter of a socket plunger as 20.47 and 20.65 mm; and the target value of 20.56 mm. The manufacturing company determined that for this process to be capable, $C_{pm} \geqq 1.0$ (i.e the required capable level $C_0$ is 1.0) and $1 - \alpha = 0.95$. They took random sample size n = 50 and measured the inner diameter of the socket plunger in mm, as follows :

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 20.54 | 20.55 | 20.54 | 20.55 | 20.53 | 20.59 | 20.51 | 20.56 | 20.54 | 20.54 |
| 20.63 | 20.54 | 20.55 | 20.54 | 20.56 | 20.55 | 20.54 | 20.61 | 20.54 | 20.57 |
| 20.56 | 20.57 | 20.58 | 20.57 | 20.58 | 20.57 | 20.57 | 20.59 | 20.56 | 20.55 |
| 20.51 | 20.55 | 20.54 | 20.60 | 20.53 | 20.54 | 20.54 | 20.55 | 20.57 | 20.56 |

The samples give the values $\overline{X}$=20.5578, S=0.0231, $\widehat{C_{pm}}$ =1.2901, and the RMV $C^*$ is 1.0138. Since, $\widehat{C_{pm}} > C^*$, therefore, we conclude that the process is capable with a 95% confidence level.

The RMV approach can be used to compare the values of RMV and $\widehat{C_{pm}}$ in order to test whether the production process is capable or not. From the tables provided by Cheng(1992), we can't find the rejected probability at $\widehat{C_{pm}} = 1.2901$. It can only be replaced by an approximate value. Besides, when using the Cheng's graphic method, we can't know what is the lower limit of capable quality level in production line at $C = C_0$. However, the RMV approach provided in this paper is able to figure out that lower limit. Through the above simple example, we can know that the RMV approach developed in this paper is better than Cheng's graphic method.

# The MPPAC of $C_{pp}$

Singhal (1990) proposed a graphic chart (multiprocess performance analysis chart; MPPAC) which was based on the process capability index. Greenwich and Jahr-Schaffrath (1995) proposed the $C_{pp}$ index based on incapability to evaluate product process, where $C_{pp}$ is an incapability index converted from $C_{pm}$, $C_{pp}=\left(\frac{1}{C_{pm}}\right)^2$. The higher the $C_{pp}$ means the worse the process capability and vice versa. $C_{pp}$ is defined as following :

$$C_{pp}=\left(\frac{1}{C_{pm}}\right)^2=\left(\frac{\mu-T}{d}\right)^2+\left(\frac{\sigma}{d}\right)^2,\ \text{where } d=\frac{USL-LSL}{6} \qquad (9)$$

Let the first term of above equation $\left(\frac{\mu-T}{d}\right)^2$ be $C_{ia}$ (called inaccuracy index) , and the second term $\left(\frac{\sigma}{d}\right)^2$ is $C_{ip}$, called imprecision index. In other words, the reasons for variation among product processes are divided into the departure of process mean from target value and the process variability.

Let the $C_{pp}$ be equal to k, then the equation (9) becomes :

$$kd^2=(\mu-T)^2+\sigma^2 \qquad (10)$$

It is easy to find that equation (10) is a semicircle with (T,0) at the center and with a radius of $\sqrt{k}\,d$. If the process capability is better, the k value will be smaller, and so will be the radius of the semicircle.

Since $C_{pp}=(1/C_{pm})^2$, we can compute the $C_{pp}$ values if the $C_{pm}$ values are given as following :

| $C_{pm}$ | 0.30 | 0.50 | 1.00 | 1.30 | 1.50 | 1.80 |
|---|---|---|---|---|---|---|
| $C_{pp}$ | 11.11 | 4.00 | 1.00 | 0.59 | 0.44 | 0.31 |

Then, a few different semicircles having the same center with above values can be drawn as figure 1.

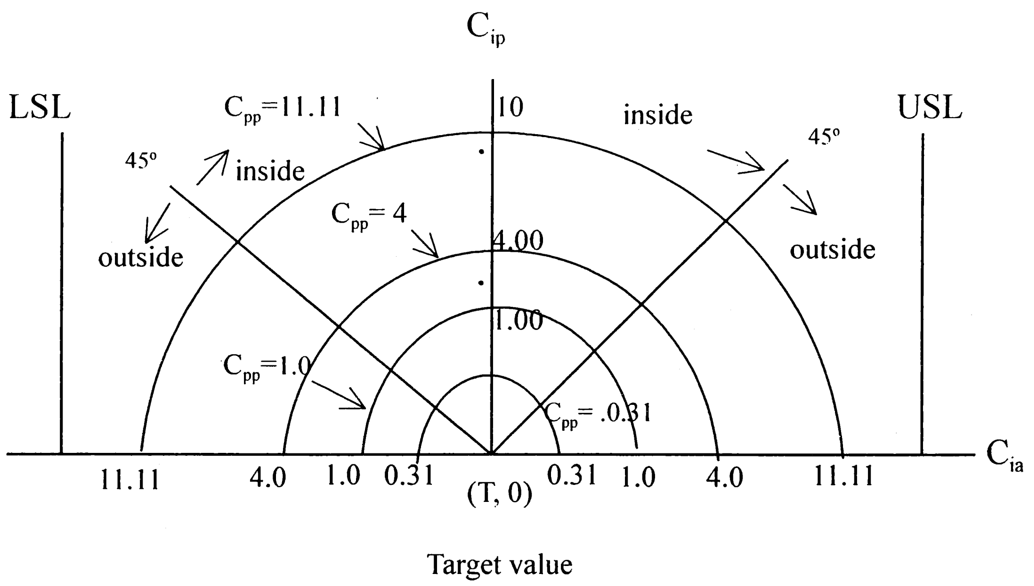


**Figure 1. A graphic demonstration of MPPAC of $C_{pp}$**

There are 5 characteristics in the MPPAC of $C_{pp}$ index:

1. The smaller $C_{pp}$ value indicates a smaller semicircle and better process capability.
2. The distance from point (T, 0) to any point which is the intersection of perpendicular line and horizontal axis represents the departure of the process mean from the target value ( $\overline{X} - T$ ).
3. The distance from any point which is the intersection of horizontal line and vertical axis to the horizontal axis represents the standard deviation of the process (S).
4. For a fixed $C_{pp}$ value, the area enclosed by two 45° lines indicates a process variation which is caused by a process variability (Sn) which is bigger than the departure of process mean from target value ( $\overline{X} - T$ ). The areas outside two 45° lines indicate the process variation of the departure of process mean from

target is bigger than the process variability.

5. The index value of $C_{ia}$ represents the distance from (T,0) to the intersection of both the horizontal axis and a prependicular line drawn from any point on the semicircle.
6. The index $C_{ip}$ value represents the distance from (T,0) to the intersection of both the vertical axis and a horizontal line drawn from any point on the semicircle.

Practically, the MPPAC of $C_{pp}$ has the following advantages :

1. It can be used to simultaneously evaluate the quality of multiprocesses.
2. It can be used to prioritize quality improvements.
3. It can make the direction of the process improvement effectively and easily understood from the MPPAC.
4. It provides uncontaminated separation between information concerning process accuracy and process precession.

# NUMERICAL EXAMPLES

A testing procedure for RMV of $\widehat{C}_{pm}$ and the MPPAC of $C_{pp}$ was explained in the previous sections. In this section, the testing procedure and the graphical chart of MPPAC of $C_{pp}$ is illustrated using the following examples.

**Example 1** : To test if the foot length of IC meet the quality demand

This example used the foot length of IC as a quality characteristic to test whether the production process of IC foot is capable or not. The quality characteristic data were collected from June 11, 1998 to December 12, 1998 with a total of 2900 sample. The data are shown in Table 1-(c). Figure 2 demonstrates how the quality characteristics of IC are tested.

**Table 1 The sample data of several quality characteristics from IC process**

(a)Quality char. : tensile strength of golden wire

| No. | Class Interval | Frequency |
|---|---|---|
| 1 | 6.800-7.423 | 15 |
| 2 | 7.423-8.046 | 21 |
| 3 | 8.046-8.669 | 63 |
| 4 | 8.669-9.292 | 149 |
| 5 | 9.292-9.915 | 232 |
| 6 | 9.915-10.538 | 336 |
| 7 | 10.538-11.162 | 374 |
| 8 | 11.162-11.785 | 328 |
| 9 | 11.785-12.408 | 292 |
| 10 | 12.408-13.031 | 147 |
| 11 | 13.031-13.654 | 71 |
| 12 | 13.654-14.277 | 31 |
| 13 | 14.277-14.900 | 21 |

Total sample number : 2080
Sample mean : 10.9236
Sample SD : 1.3815

(b)Quality char. : thickness of tin coating

| No. | Class Interval | Frequency |
|---|---|---|
| 1 | 328.328-363.172 | 7 |
| 2 | 363.172-398.015 | 28 |
| 3 | 398.015-432.859 | 76 |
| 4 | 432.859-467.703 | 148 |
| 5 | 467.703-502.547 | 211 |
| 6 | 502.547-537.390 | 185 |
| 7 | 537.390-572.234 | 159 |
| 8 | 572.234-607.078 | 88 |
| 9 | 607.078-641.922 | 37 |
| 10 | 641.922-676.765 | 28 |
| 11 | 676.765-711.609 | 8 |

Total sample number : 975
Sample mean : 491.948
Sample SD : 85.570

(c)Quality char. : foot length

| No. | Class Interval | Frequency |
|---|---|---|
| 1 | 17.000-17.862 | 6 |
| 2 | 17.862-18.723 | 23 |
| 3 | 18.723-19.585 | 72 |
| 4 | 19.585-20.446 | 189 |
| 5 | 20.446-21.308 | 325 |
| 6 | 21.308-22.169 | 492 |
| 7 | 22.169-23.031 | 595 |
| 8 | 23.031-23.892 | 529 |
| 9 | 23.892-24.754 | 415 |
| 10 | 24.754-25.615 | 175 |
| 11 | 25.615-26.477 | 61 |
| 12 | 26.477-27.338 | 17 |
| 13 | 27.338-28.200 | 1 |

Total sample number : 2900
Sample mean : 22.626
Sample SD : 2.244

(d)Quality char. : coplanarity of two legs

| No. | Class Interval | Frequency |
|---|---|---|
| 1 | 0.050-.0238 | 20 |
| 2 | 0.238-0.427 | 81 |
| 3 | 0.427-0.615 | 194 |
| 4 | 0.615-0.804 | 701 |
| 5 | 0.804-0.992 | 935 |
| 6 | 0.992-1.182 | 994 |
| 7 | 1.182-1.369 | 271 |
| 8 | 1.369-1.558 | 36 |
| 9 | 1.558-1.746 | 11 |
| 10 | 1.746-1.935 | 9 |
| 11 | 1.935-2.123 | 5 |
| 12 | 2.123-2.312 | 1 |
| 13 | 2.312-2.500 | 2 |

Total sample number : 3260
Sample mean : 0.9202
Sample SD : 0.2251

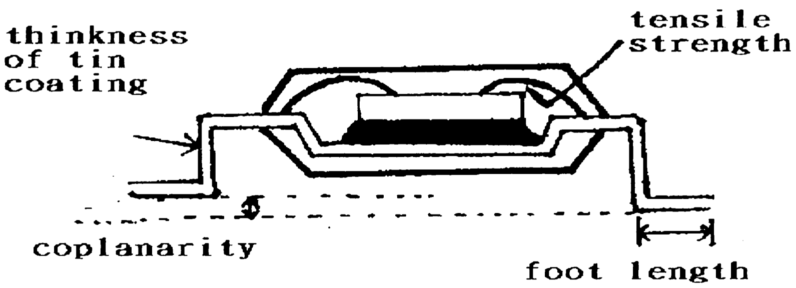


Figure 2. The four quality characteristics of IC are tested

The population of quality characteristic is assumed from normal distribution when deduate the equation(8), the test of normal distribution of population of IC foot length and the test procedures using RMV approach are as followed :

**Step 1 : $H_0$** : Foot length is from Normal distribution.

**$H_1$** : Foot length is not from Normal distribution.

**Step 2 :** The Chi-Square test of Goodness of fit is used to test the above hypothesis, and calculations for testing the hypothesis as following :

| No. | Probability | Expect freq. ($e_i$) | Observed Freq. ($f_i$) | $e_i - f_i$ | $(e_i - f_i)^2/ e_i$ |
|---|---|---|---|---|---|
| 1 | 0.001652482 | 4.77567376 | 6 | -1.22432624 | 0.313877124 |
| 2 | 0.006472279 | 18.7048856 | 23 | -4.2951144 | 0.986266802 |
| 3 | 0.022445914 | 64.8686923 | 72 | -7.13130774 | 0.783976805 |
| 4 | 0.059342831 | 171.500781 | 191 | -19.4992192 | 2.217013528 |
| 5 | 0.119214369 | 344.529528 | 320 | 24.5295276 | 1.746432964 |
| 6 | 0.181130126 | 523.466065 | 493 | 30.4660654 | 1.773144814 |
| 7 | 0.209626549 | 605.820728 | 596 | 9.82072803 | 0.159200065 |
| 8 | 0.183583318 | 530.555789 | 528 | 2.55578878 | 0.012311724 |
| 9 | 0.122389855 | 353.706681 | 410 | -56.2933188 | 8.959225016 |
| 10 | 0.061478631 | 177.673243 | 173 | 4.67324329 | 0.122917793 |
| 11 | 0.022299044 | 64.4442374 | 60 | 4.44423743 | 0.306485842 |
| 12 | 0.008918501 | 25.774469 | 17 | 8.77446901 | 2.987115132 |
| 13 | 0.001165273 | 3.36763867 | 1 | 2.36763867 | 1.664582639 |
| Total | 0.999719174 | 2,889.1884 | 2,890 | -0.8116 | 22.032550248 |

**Step 3 :** Since the test statistic $\chi^2_{0.01}(13\text{-}2\text{-}1)$ = 23.2093. So, we are unable to reject the hypothesis $H_0$.

**Step 4 :** Consider Table 1-(c), we obtain the sample mean $\bar{x}$=22.6257 and sample standard deviation s = 2.2443.

**Step 5 :** The specification limits, target value, the required process capability value and significance level are provided as following : USL = 28, LSL = 15, T = 21.5, $C_0$ = 1.6, $\alpha$ = 0.01.

**Step 6 :** A total of 2900 observations are measured and the $\widehat{C_{pm}}$ = $\dfrac{USL-LSL}{6\sqrt{s^2+(\bar{x}-T)^2}}$ are calculated. We obtain $\widehat{C_{pm}}$ = 1.893.

**Step 7 :** Calculated the recommended minimum value $C^*$. Calculation yields $C^*$ = 1.743.

**Step 8 :** Make a decision :

Since $\widehat{C_{pm}}$ = 1.893 > $C^*$ = 1.743, therefore, we conclude that the process of foot length is capable with a 99% confidence level.

**Example 2 : The application of MPPAC of** $C_{pp}$ in IC production

The following example uses MPPAC of $C_{pp}$ index to analyze the process capabilities of multiple quality characteristics simultaneously.

In the process of IC production, many quality characteristics (such as adhesive strength of golden wire, thickness of tin coating, coplanarity and so on) need to be monitored at the same time. The MPPAC of $C_{pp}$ developed in this paper is capable of monitoring several quality characteristics in IC production process. There are four quality characteristics in this example : tensile strength of golden wire, thickness of tin coating, foot length and coplanarity (co – plane tolerance of two foot). The tensile strength of golden wire is to test if the adhesive strength of golden wire meets the quality demand. In order to do so, a WX – 22 type test machine is used to pull the golden wire between die and frame. The pulling force when the golden wire breaks is then recorded. The thickness of tin coating is to test the thickness of tin coating on the IC leg. The foot length is the length of an IC in level direction. The coplanarity is the elevation difference tolerance between left and right foot of an IC.

The four quality characteristics data were collected from June 11, 1998 to Dec. 12, 1998, the data are shown in Table 1. The upper, lower specifications, target value, sample mean, sample standard deviation and the inaccuracy index $C_{ia}$, imprecision index $C_{ip}$ of the four quality characteristics are shown in Table 2.

**Table 2 Results of sample statistics, inaccuracy index and imprecision index**

| Quality chart. | USL | LSL | T | n | $\overline{X}$ | S | $\widehat{C}_{ia}$ | $\widehat{C}_{ip}$ | $\widehat{C}_{pp}$ |
|---|---|---|---|---|---|---|---|---|---|
| Tensile strength of wire (g) | 16.0 | 4.0 | 12.0 | 2080 | 10.9236 | 1.3815 | 0.2897 | 0.4771 | 0.7668 |
| Thickness coating of tin (m $\mu$ inch) | 1000 | 300 | 650 | 975 | 491.9481 | 85.5704 | 1.8353 | 0.5379 | 2.3733 |
| Foot length ($\mu$ inch) | 28 | 15 | 21.5 | 2900 | 22.6257 | 2.2443 | 0.2699 | 1.0359 | 1.3058 |
| Coplanarity ($\mu$ inch) | 3.5 | 0.0000 | 0.009 | 3260 | 0.9202 | 0.2251 | 2.4401 | 0.1489 | 2.5889 |

Let $\left(\frac{\overline{X}-T}{d}\right)^2 = \widehat{C}_{ia}$ to reflect the process inaccuracy (deviation of the process mean departure from target value) and $\left(\frac{S}{d}\right)^2 = \widehat{C}_{ip}$ to reflect the process imprecision (process variation). To illustrate the calculation of estimators, we consider the coplanarity characteristic, assuming the target value T = 0.009, USL = 3.5, LSL = 0.00, and the sample mean $\overline{X}$ = 0.9202, sample standard deviation S =

0.2251. Then, we can calculate $\widehat{C_{ia}} = \left(\frac{\frac{0.9202-0.009}{6}}{\frac{3.5-0}{6}}\right)^2 = 2.44$, $\widehat{C_{ip}} = \left(\frac{0.2251}{\frac{3.5-0}{6}}\right)^2 = 0.1489$, and $\widehat{C_{pp}} = \widehat{C_{ia}} + \widehat{C_{ip}} = 2.44 + 0.1489 = 2.5889$.

From Table 2, we can see that the target values of the quality characteristic of tensile strength of wire, thickness of tin coating and coplanarity do not equal to $\frac{USL-LSL}{6}$. Coplanarity possesses high inaccuracy process; nevertheless, tensile strength of wire possesses low inaccuracy process and low imprecision process. Thus, the incapability index $C_{pp}$ is not concerned with whether T equals $\frac{USL-LSL}{2}$ or not.

Using the $C_{ia}$ and $C_{ip}$ data from Table 2 to draw the MPPAC of $C_{pp}$, as shown in Figure 3, we may easily and simultaneously evaluate the four quality characteristics of IC production process from the positions of every point in the MPPAC. From Figure 3, we can see tensile strength of wire has the best process quality, on the other hand, the coplanarity has the worst process. We conclude the process status and the direction of improvement of the four processes as follows :

1. Point A is the nearest to center ($C_{pp}$ = 0.7668), and its $C_{ia}$ = 0.2897 and $C_{ip}$ = 0.4771 are both very small. This implies that the process of tensile strength of wire has the best process capability among the four process.
2. Point B is located outside the contour line of $C_{pp}$ = 2 and its $C_{ia}$ value(1.8353) is greater than its $C_{ip}$ value(0.5379), which means the deviation of the process mean departure from target is greater than the degree of process variability. Hence, the deviation of the process mean departure from target of the thickness of tin coating process must be improved.

3. Point C is located outside the contour line of $C_{pp}$= 1 and inside the two 45° lines (representing $C_{ia} < C_{ip}$), which implies the process variability is greater than the deviation of the process mean departure from target value. Hence, the improvement focus for foot length process is to reduce its process variability.
4. Point D is located outside the contour line of $C_{pp}$= 2 and its $C_{ia}$ values(2.4401) is more greater than $C_{ip}$ value(0.1489), which implies the process of coplanarity possess lower process variability and its deviation of process mean departure from target is very large. So, the improvement focus for coplanarity possess is to reduce the deviation of the process mean departure from target.

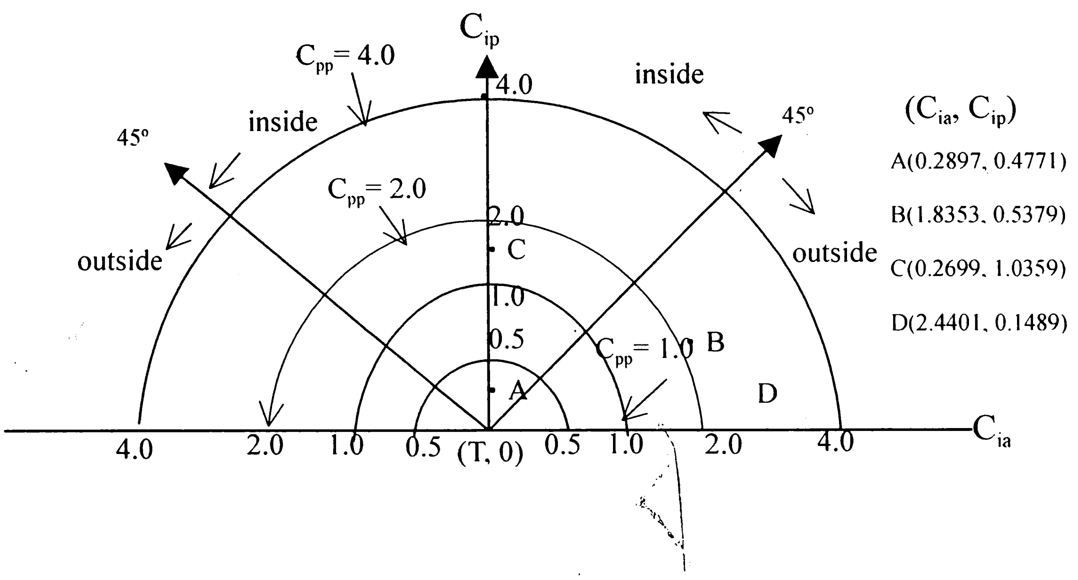


**Figure 3. The graphic illustration of MPPAC for four quality characteristics of IC**

# CONCLUSION

The $C_{pm}$ index can be used to evaluate the ability of a process to attain a preset target value and to concurrently fall within required specification limits. Boyles (1991) made a comparison among $C_p$ , $C_{pk}$ , $C_{pm}$ and concluded that $C_{pm}$ is the most useful index because $\widehat{C_{pm}}$ is less biased and more efficient than $\widehat{C_p}$ , $\widehat{C_{pk}}$. Therefore, in this paper, the $C_{pm}$ index is applied to evaluate the process performance. First, we derive the recommended minimum values of $\widehat{C_{pm}}$ and propose a testing procedure for determining whether a process is capable or not. Since the $C_{pp}$ index, which is transformed from the $C_{pm}$ index, has advantages in simultaneously evaluating the accuracy and precision of process. The $C_{pp}$ index is then applied to develop a MPPAC of $C_{pp}$ in this paper. This chart can analyze process capabilities of multiple processes simultaneously, thus making the information for evaluating the process more clear. In conclusion, a testing procedure for the RMV of $\widehat{C_{pm}}$ is offered to accurately assess single process capability, and a MPPAC of $\widehat{C_{pp}}$ is applied to indicate the effective direction for quality improvement when evaluating the multiple processes.

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# Appendix I : Derivation of Equation (5)

Since $\frac{\widehat{n\tau^2}}{\sigma^2} \sim \chi'^2_{n,\sqrt{n}|\xi|}$ (Proved in Thm.1) and the mean and variance of this distribution are given by $n(1+\xi^2)$ and $2n(1+2\xi^2)$, respectively (Scheffe, 1959).

$$\text{i.e.} \begin{cases} E(\frac{\widehat{n\tau^2}}{\sigma^2}) = n + n\xi^2 \\ Var(\frac{\widehat{n\tau^2}}{\sigma^2}) = 2n + 4n\xi^2 \end{cases}$$

This distribution is well approximated by a Chi-square distribution of the form $c \cdot \chi^2_k$, where c is a constant, k is the degree of freedom (Scheffe, 1959).

By solving the equations :

$$\begin{cases} ck = n(1+\xi^2) \\ c^2k = n(1+2\xi^2) \end{cases}$$

we get

$$c = \frac{(1+2\xi^2)}{1+\xi^2}$$

$$k = \frac{n(1+\xi^2)^2}{(1+2\xi^2)}$$

i.e. $\sqrt{\frac{\widehat{n\tau^2}}{\sigma^2}} \sim \frac{1+2\xi^2}{1+\xi^2}\chi^2_k$, and $\sqrt{\frac{\widehat{n\tau^2}}{\sigma^2}} = \sqrt{\frac{n(1+\xi^2)\widehat{\tau^2}}{(1+\xi^2)\sigma^2}} = \sqrt{\frac{n(1+\xi^2)\widehat{\tau^2}}{\tau^2}}$

Thus,

$$p_r\left\{\widehat{C_{pm}}\sqrt{\frac{1}{n(1+\xi^2)}\chi'^2_{n,\sqrt{n}|\xi|}} \geq c\right\} \quad \text{(Eq. 5)}$$

$$= p_r\left\{\hat{C}_{pm}\sqrt{\frac{1}{n(1+\xi^2)} \cdot \frac{1+2\xi^2}{1+\xi^2} \cdot \chi^2_k} \geq c\right\}$$

$$= p_r\left\{\hat{C}_{pm} \cdot \sqrt{\frac{1+2\xi^2}{n(1+\xi^2)^2} \cdot \chi^2_k} \geq c\right\} \quad \text{(Eq. 6)}$$

Hence, equation (5) can be approximated by equation (6).