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Optimal Allocation of Capacitated Facilities Considering Time-Dependent User Preference for User Number Maximization

最大化偏好因時而異之消費者服務人數的有限產能設施選址模型

Ling-Chieh Kung, Department of Information Management, National Taiwan University
孔令傑 / 國立臺灣大學資訊管理學系

Jih-Sheng Chuang, Department of Information Management, National Taiwan University
莊日陞 / 國立臺灣大學資訊管理學系

Yun-Tung Kuo, Department of Information Management, National Taiwan University
郭芸彤 / 國立臺灣大學資訊管理學系

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Abstract

Traditional facility location models assume that the decision maker may assign a facility to serve a customer. While this assumption can be true in the case of assigning distribution centers to retail stores, it does not apply to the scenarios where end consumers choose serving facilities according to personal preferences. The problem becomes even more challenging when the facility is with limited capacity and the customer is time-dependent. In this study, we consider a decision maker who builds facilities of various scale levels to maximize the number of customers served. We propose a mixed integer programming formulation to describe the problem. As the problem is NP-hard, we develop a heuristic algorithm by reducing part of the problem to the maximum flow problem. Through numerical studies we demonstrate the effectiveness of our proposed algorithm.

【Keywords】facility location, preference, capacity, time dependency, maximum flow

摘 要

在傳統的設施選址問題中，決策者可以指定由哪個設施去服務哪個顧客。雖然這可能適用於配送中心與零售店之間；但對於根據自身偏好決定要前往哪個設施的終端消費者而言，這個設定便顯得不夠實際。當要設置的設施有容量限制，而消費者的偏好又會因時而異時，這樣的問題將變得更具挑戰性。在這個研究中，我們考慮如何建造不同規模的設施以吸引盡量多的使用者，並為此建立了一個混合整數規劃模型；又由於此問題是 NP-hard，本研究開發了一個將一部份問題轉換為最大流問題的啟發性演算法，並利用數值實驗來檢驗此演算法的成效。

【關鍵字】設施選址、偏好、容量限制、因時而異、最大流

領域主編：蔡瑞煌教授

1. Introduction

The facility location problems have been widely discussed for decades. In a typical facility location problem, a decision maker decides where to build facilities among some given locations. Typical objectives of the problem include profit maximization, cost minimization, etc. for the decision maker. Some facility location problems are uncapacitated, i.e., facilities are not subject to capacity constraints. On the contrary, there may be capacitated facility location problem, in which facilities have limited amount of capacity. Applications of facility location problems lie in the fields of supply chain management (Pirkul and Jayaraman, 1998; Shavarani, Mosallaeipour, Golabi, and İzbirak, 2019), logistics (Lu and Bostel, 2007; Hosseini, Dehghanian, and Salari, 2019), operations management (Harkness and ReVelle, 2003; Filippi, Guastaroba, and Speranza, 2021), and healthcare (de Vries, van de Klundert, and Wagelmans, 2020).

When it comes to service facilities, several extensions for the facility location problem have been studied in recent years. One extension is to consider customers, which is critical when one builds service facilities like retail stores, parks, hospitals, public bike stations, etc. Naturally, these service facilities are heterogeneous to customers, i.e., customers would have different preferences toward them. In this case, whether one customer would visit one specific facility or not cannot be determined directly by the decision maker. Technically speaking, in a traditional facility location problem, a customer can be "assigned" to a facility (e.g., a retail store can be assigned to a warehouse for replenishment), but in a service facility location problem, a customer cannot be "assigned" to a facility (e.g., a retail chain owner cannot require a citizen to visit any specific retail store) and can only be "attracted" to a facility. Each of the customers will choose facilities to visit based on her/his preference. Applications of facility location problem with customer can be in many industries. For example, Shih, Chang, and Peng (2002) verify the relationship among customers' pick-up demands, chain store locations, and store performances with empirical studies. Hiassat (2017) presents a model for deciding facility locations and allocating healthcare resources according to different customer types. Furthermore, the relative problems are also studied for emergency response facilities (Li, Zhao, Zhu, and Wyatt, 2011; Abdullah, Adawiyah, and Kamal, 2018), retail stores (Hanjoul and Petters, 1987), power stations (Abdel-Basset, Gamal, Chakrabortty, and Ryan, 2021), among others.

A further extension for customer is to incorporate the time dependency of preference. In many cases, once a service facility is built, it is open to customers throughout a day. However, for a single facility, a customer may have different preferences over different activity sessions in a day. For example, while several citizens may be willing to exercise in a particular park, some may prefer to exercise in the morning while some may prefer to do so in the evening. Suppose that a day is split into three activity sessions: morning, afternoon, and evening. To calculate the total number of citizens that may exercise in the park in a day, we cannot just multiply that number in any one of the three sessions by three. Instead, a model with three activity sessions must be constructed to correctly estimate the benefit of building that park. In short, taking the time factor into account is needed to make a model much closer to reality.

One major motivating applications of our proposed model is to build public sport facilities. With medical technology progressing and economic growth in recent years, population aging has become a worldwide issue to be addressed. To improve the welfare of an aging society, it is suggested for a government to increase the frequency and strength of regular exercise of the elder (Laforge, Rossi, Prochaska, Velicer, Levesque, and McHorney, 1999). To make this happen, constructing enough public sport facilities that are appropriate to the elder is crucial. To make a good construction decision, the first step is being able to estimate the benefit of building some facilities, which may be measured by the number of elders using built facilities. An important feature of this problem is that the government cannot specify a facility for a citizen; instead, each citizen will make her/his own choice. Whether a construction plan may really benefit citizens cannot be determined if customer is ignored. Note that typically one citizen only goes out for exercise once in a day, different elders prefer different time for exercise, and a facility is generally open throughout a day. Therefore, if the government neglects the elders' time preferences and assumes that all elders will compete for the capacitated facility at the same time, the effective capacity of a facility becomes underestimated. Formulating a model that includes time-dependent customer and multiple activity sessions in a day is thus required for this construction plan. In addition, to the best of our knowledge, no previous study simultaneously takes aforementioned factors into account. Therefore, our investigation and consideration of a capacitated facility location problem along with customers' time-dependent preferences contributes to the literature.

To model time dependency, it is assumed that one day is split into multiple non-overlapping activity sessions (e.g., morning, afternoon, evening, and midnight). There are groups of customers with different population sizes locating at various places. A decision maker plans to choose the locations and scale levels to build facilities from a set of candidate locations. The scale level determines the number of customers that may be served at this facility in an activity session. Customers in the same group tend to have identical preferences over each facility in each activity session. Still, they may hold different preferences over different facilities or different activity sessions. That is, among all built facilities that are still having rooms in some activity sessions, a customer will choose her/his most preferred facility-session pair to visit that facility in that activity session. Nonetheless, we also assume that a customer always has the option of staying at home without visiting any facility, which gives her/him a null utility. That is, if one finds that no facility-session pair may give her/him a positive utility, she/he will choose to stay at home.

The decision maker acts to maximize the number of served customers subject to a budget constraint. More specifically, the decision maker builds several facilities with a constraint that the total construction cost cannot exceed the budget limit. Each of the customers then self-selects among built facilities subject to the capacity constraint or chooses to stay at home without visiting any facility. The objective function of the decision maker is to maximize the number of customers who visit a facility.

To solve the aforementioned decision maker's problem of building capacitated service facilities, we develop two different solution approaches. The first one is through formulating a mixed integer program so that when one seeks for an exact solution, she/he may obtain it by solving the program using an existing exact algorithm (such as branch and bound). Kang, Kung, Chiang, and Yu (2023) proposes a bi-level model that incorporates preference and capacity based on Hanjoul and Petters' (1987) formulation. Nevertheless, Kang et al. (2023)'s model has many constraints and high time complexity also causing the problem becomes harder to be solved in acceptable time. Camacho-Vallejo, Casas-Ramírez, and Miranda-Gonzalez (2014) propose another formulation to turn a bi-level model into a single-level one; however, the capacity issue is omitted in their formulation. Combining these two works, we formulate a single-level mixed integer program for our capacitated facility location problem considering customer. We further extend the model to incorporate multiple activity sessions and allow preferences to be time-dependent.

Given that the problem is NP-hard, in many cases a heuristic algorithm is more realistic. In Kang et al. (2023), a heuristic greedy algorithm is designed based on the maximum flow model. However, there are two drawbacks of this algorithm. First, the time dependency of preference is ignored. Second, the algorithm is computationally inefficient due to the fact that the benefits of two neighboring solutions are evaluated by solving two independent problems. In this study, we extend the algorithm to take time dependency into consideration and speed it up by utilizing some properties among neighboring solutions.

Choosing locations to build facilities is difficult in practice, especially when customers' preference is time-dependent. To take time-dependent preference into consideration, for each construction plan, one needs to be able to calculate (or at least estimate) the number of customers that may be served in each activity sessions. However, even considering capacity limitation and customer together is already difficult enough (given the fact that there is almost no academic literature studying these two issues at the same time; see Section 2 for more details), not to mention time dependency. Two simplified strategies that practitioners typically adopt are: (1) to consider only the activity session that has the most demand (if demand varies a lot among sessions); (2) to consider the average demand per session (if demand variation is not large). However, these strategies make the evaluation of a construction plan imprecise in general. On the contrary, the maximum flow-based algorithm proposed in this study allows one to precisely calculate the number of customers that may be served in each activity session for any construction plan. Our proposed algorithm therefore possesses a unique value.

The remainder of this study is organized as follows. In the next section, we review some related works. In Section 3, a bi-level model is presented first. Then a single-level reformulation is proposed by using auxiliary variables and constraints. In Section 4, we propose a greedy-based heuristic algorithm for solving the problem. A numerical study is conducted to demonstrate and compare the performance of the two proposed solution approaches in Section 5. Finally, we make conclusions in Section 6.

2. Literature Review

A facility location problem is a problem where a decision maker decides where to locate facilities and how to assign customers to those built. While many facility location problems take facility capacity into consideration (see, e.g., Ho (2015) and Gadegaard, Klose, and Nielsen (2018) and the references therein), none of these works include customers' self-selection based on their preferences.

Traditionally, to put customer into a model, researchers usually formulate the problem as a bi-level program. In such a program, the upper-level problem is for the decision maker to decide locations to build facilities, and the lower-level one is for customers to choose facilities to visit. To solve these bi-level programs, reformulating them into single-level programs is typical. Three reformulations of the uncapacitated bi-level problem using sets to express customer are proposed by Hanjoul and Petters (1987) where two greedy-based heuristic algorithms are also presented. Some other researchers, e.g., Hansen, Kochetov, and Mladenovi (2004), Cánovas, García, Labbé, and Marín (2007), and Vasil'ev, Klimentova, and Kochetov (2009), reformulate the model using similar ideas. More recently, Camacho-Vallejo, Cordero-Franco, and González-Ramírez (2014) present two reformulations. Using the primal-dual relationship and complementary slackness of the lower level, they obtain two linearized single-level facility location models.

As the reformulation methods adopted in the above works prove to be not so efficient, researchers have invented other approaches. In particular, Berman, Drezner, Tamir, and Wesolowsky (2009), and Espejo, Marín, and Rodríguez-Chía (2012) both propose the so-called "closest assignment constraints" to turn the bi-level model into a single-level one; Camacho-Vallejo, Casas-Ramírez, and Miranda-Gonzalez (2014) further apply this idea to facility location problems with customers. Their computational result shows that the new reformulation requires less time compared to those previous reformulation methods. Thus, in this study, we borrow the idea from Camacho-Vallejo, Casas-Ramírez, and Miranda-Gonzalez (2014) and add capacity and time-dependent preference constraints to formulate our model.

There are some more recent works regarding customers in facility location problems. Based on the models designed by Camacho-Vallejo, Cordero-Franco, and González-Ramírez (2014) and Camacho-Vallejo, Casas-Ramírez, and Miranda-Gonzalez (2014), Casas-Ramírez, Camacho-Vallejo, and Martínez-Salazar (2018) use a cross entropy method to solve the upper-level problem and a greedy randomized adaptive procedure to solve the lower-level one. Drezner, Drezner, and Zerom (2018), though do not directly model customers, assume that the facilities' attractiveness may be randomly distributed. Calvete, Galé, Iranzo, Camacho-Vallejo, and Casas-Ramírez (2020) add the cardinality constraint into a facility location problem with preference by limiting the maximum number of customer points that can be assigned to each facility point. Notably, these works either ignore the capacity issue or only impose a weaker version of the capacity constraint (e.g., the cardinality constraint). We contribute to the literature by incorporating the capacity and preference issues in a single model.

Our goal is to build several finite-capacity facilities under a budget constraint and to maximize the total number of customers with time-dependent preferences. To the best of our knowledge, Kang et al. (2023) is so far the only work that explicitly includes both customer and facility capacity in a single model (though excludes customers' time preferences). He proposes a greedy algorithm for solving that NP-hard problem. In each iteration, the benefit evaluation problem is transformed into a maximum flow problem, and the location with the highest benefit-to-cost ratio is selected. In our study, we extend the formulation and revise the algorithm to incorporate the time factor.

3. Problem Description and Formulation

In this section, we provide the statement and formulation of our capacitated facility location problem with time-dependent user preference.

3.1 Uncapacitated Facilities with Customer Preference

We consider a decision maker deciding where to build facilities along with the scale levels but without capacity constraints. Let $J=\{1,2,3,...,|J|\}$ denote the set of locations where a facility may be built, and $K=\{1,2,3,...,|K|\}$ represent the set of scale levels that for each facility decision maker may choose from. The parameter f_{jk} represents the fixed cost of building the facility at location j with scale level k. Without loss of generality, we assume that $0 \leq f_{j,1} \leq f_{j,2} \leq \ldots \leq f_{j,|K|}$ for all facility j.[1] For ease of exposition, we may call the facility built on location j as facility j from time to time. $I=\{1,2,3,\ldots,|I|\}$ is the set of customer locations, where d_i is the population size at customer location i. For ease of exposition, we may call the customers at location i as customer i, and d_i as the demand of customer i from time to time. It is assumed that one day is split into several non-overlapping activity sessions in which one customer chooses at most one session to visit a facility. To represent the fact, let $T=\{1,2,3,\ldots,|T|\}$ be the set of activity sessions. Customer i has a preference level over facility j in activity session t, represented by p_{ijt}. We have $p_{i,j1,t}>p_{i,j2,t}$ if customer i prefers facility j_1 to facility j_2 in activity session t. For those customers at the same location, we assume that they have identical preference for the same facility in the same activity session. The total budget for building facilities is B.

The decision maker's decision is to choose locations to build facilities at a certain scale level. To model this, let $y_{jk} \in \{0,1\}$ be 1 if a facility is built at location j with scale level k or 0 otherwise. The special case $y_{0,k}$ for any k is always 1 since customers can decide to stay at home in any activity sessions. After facilities are built, each customer either chooses one activity session to visit one facility or stays at home. That decision is made according to her/his preferences. We assume that customers' preferences are exogenous; i.e., the preference over one facility will not be affected by other customers' decisions or whether other facilities are built or not.

We first model customers' choice when facilities all have ample capacity, i.e., the number of customers going to the same facility in the same activity session is unlimited. In this case, let $x_{ijt} \in \{0,1\}$ present whether customers at location i go to facility j in activity session t ($x_{ijt}=1$) or not ($x_{ijt}=0$). Note that x_{ijt} will not be fractional in equilibrium, i.e., all customers at the same location may make the same decisions, because the capacity of facility is infinite.

Collectively, we may formulate the decision maker's problem as[2]

1 If the number of the scale level candidates of location j is less than $|K|$, set $f_{j,k'}$ to infinite for those $k' \in K$ which cannot be chosen for location j.

2 Note that in this formulation, it does not matter whether we set $x_{ijt} \in \{0,1\}$ or $x_{ijt} \in [0,1]$. However, as we do not intend to solve this uncapacitated problem, we leave the setting to be binary to highlight the fact that x_{ijt} will either be 0 or 1 in an optimal solution.

$$\max \quad \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} d_i x_{ijt}, \tag{1}$$

$$\text{s.t.} \quad \sum_{k \in K} y_{jk} \leq 1 \quad \forall j \in J, \tag{2}$$

$$\sum_{j \in J} \sum_{k \in K} f_{jk} y_{jk} \leq B, \tag{3}$$

$$\sum_{j \in J} \sum_{t \in T} x_{ijt} \leq 1 \quad \forall i \in I, \tag{4}$$

$$x_{ijt} \leq \sum_{k \in K} y_{jk} A_{ijt} \quad \forall i \in I, j \in J, t \in T, \tag{5}$$

$$x_{ijt} \in \{0,1\} \quad \forall i \in I, j \in J, t \in T, \tag{6}$$

$$y_{jk} \in \{0,1\} \quad \forall j \in J, k \in K. \tag{7}$$

The objective function (1) of the model is to maximize the total number of served customers. Constraint (2) ensures that the decision maker can only build at most one facility with one scale level at each location. Constraint (3) requires that the total construction cost should not exceed the given budget B. In (4), the total proportion of customers visiting facilities cannot exceed 1. The fact that each customer chooses her/his most preferred facility-session pair is modeled in (5), where A_{ijt} is set to 1 if $p_{ijt} > 0$ or 0 otherwise. Constraint (5) ensures that a customer chooses a facility-session pair only if her/his possesses a positive utility over it. Note that because the decision maker cares only the total number of served customers, whether a customer goes to her/his most preferred facility-session pair or any other one is not an issue. Thus, there is no need to write this constraint to restrict one to visit her/his most preferred option. Constraints (6) and (7) state that x_{ijt} and y_{jk} are binary. Note that in this formulation, it does not matter whether x_{ijt} is fractional or binary.

3.2 Capacitated Facilities with Customer Preference

In order to incorporate capacity limitation, let q_{jk} represent the capacity of facility j with scale level k. Without loss of generality, we assume that $0 \leq q_{j,1} \leq q_{j,2} \leq \cdots \leq q_{j,|K|}$ for all facility j. Recall that it is always possible that all built facilities are not attractive enough for a customer (e.g., are all too far), and the customer may choose to stay at home without visiting any facility. To model this, we add a virtual location, location 0, into the set J' and define J={0}∪ J'. Location 0 has no construction cost (i.e., $f_{0,k}$ for all), infinite capacity (i.e., $f_{0,k}$=0 is infinite for all $k \in K$), and zero preference level for all customers in all activity sessions (i.e., $p_{i,0,t} = 0$ for all $i \in I$, $t \in T$).

We then modify x_{ijt} so that $x_{ijt} \in [0,1]$ presents the proportion of customer i going to facility j in activity session t. Note that now x_{ijt} must be fractional instead of binary because now facilities are capacitated, and it is possible for customers at the same location to make different decisions.

We now need to add constraints to ensure that a customer cannot go to a facility-session pair if there is another more preferred pair that is still available. To do this, we define three auxiliary variables w_{jt}, z_i, and $\bar{x}_{ijt}$. A binary variable w_{jt} is 1 if facility j is still available in equilibrium in activity session t with respect to the capacity constraint. The variable z_i represents customer i's preference of the most preferred facility-session pair among all available ones. The binary variable $\bar{x}_{ijt}$ is 1 if at least one customer in location i go to facility j in activity session t. In effect, $\bar{x}_{ijt}$ is 1 implies that either facility j is out of capacity in activity session t in equilibrium or that facility-session pair is the most preferred out of all available ones.

$$\max \quad \sum_{i \in I} \sum_{j \in J'} \sum_{t \in T} d_i x_{ijt}\,, \tag{8}$$

s.t. (2) – (4),

$$\sum_{k \in K} q_{jk} y_{jk} - \sum_{i \in I} d_i x_{ijt} \geq 0 \quad \forall j \in J, t \in T, \tag{9}$$

$$\sum_{k \in K} q_{jk} y_{jk} - \sum_{i \in I} d_i x_{ijt} \leq M w_{jt} \quad \forall j \in J, t \in T, \tag{10}$$

$$z_i \geq p_{ijt} w_{jt} \quad \forall i \in I, j \in J, t \in T, \tag{11}$$

$$p_{ijt}\bar{x}_{ijt} + M\left(1 - \bar{x}_{ijt}\right) \geq z_i \quad \forall i \in I, j \in J, t \in T, \tag{12}$$

$$\bar{x}_{ijt} \geq x_{ijt} \quad \forall i \in I, j \in J, t \in T, \tag{13}$$

$$x_{ijt} \geq 0 \quad \forall i \in I, j \in J, t \in T, \tag{14}$$

$$w_{jt} \in \{0, 1\} \quad \forall j \in J, t \in T, \tag{15}$$

$$\bar{x}_{ijt} \in \{0, 1\} \quad \forall i \in I, j \in J, k \in K, \tag{16}$$

$$y_{jk} \in \{0, 1\} \quad \forall j \in J, k \in K. \tag{17}$$

With such modifications, we propose a new model that integrates both customer and facility capacity as where M is a positive and sufficiently large constant.

The objective function (8) of the model is to maximize the total number of served customers. Note that the customers served by facility 0 are not counted since facility 0 represents customers staying at home. Constraints (2) – (4) are same as the uncapacitated ones. Constraint (9) ensures that the customers can only go to the location where a facility is built. Furthermore, the number of customers from all locations cannot exceed the capacity of the facility. Constraint (10) sets the value of w_{jt} to show whether facility j is still available in activity session t. If w_{jt}=0, constraints (9) and (10) together ensure that facility j is fully occupied in activity session t. On the contrary, if w_{jt}=1, constraint (10) is relaxed, and facility j still has residual capacity in the activity session t. In constraint (11), z_i is set to be the preference of the most preferred available facility for customer i. Constraint (12) sets the value of $\bar{x}_{ijt}$ according to z_i. For the facility-session pair that is most preferred by customer i, we have p_{ijt}=z_i according to constraint (11), and thus $\bar{x}_{ijt}$ can be 1. For all the remaining pairs, $\bar{x}_{ijt}$ is forced to be 0 by constraint (12). Constraint (13) then ensures that the only available facility-session pair that a customer may visit is her/his top-priority choice. The remaining constraints are nonnegativity and binary constraints.

It is worth mentioning that, as the objective function is to maximize the number of served customers, it is acceptable for a customer to visit any facility in any activity session as long as she/he obtains a positive utility (so that visiting the facility is better than staying at home). In other words, with the current objective function, a decision maker facing this problem does not need to precisely estimate all the preference levels p_{ijt}. Instead of this, the decision maker only needs to estimate whether a given preference level p_{ijt} is positive or not. In this study, we still choose to formulate the problem with p_{ijt} as a real number to retain the generality of our model.

We may show that the problem is NP-hard by reducing the problem studied in Kang et al. (2023) to our problem. This is trivial by observing that his problem is a special case of ours with $|T|=1$, i.e., there is only one activity session.

3.3 An Illustrative Example

The following example shows how these constraints express the preference relationship. In this example, there are three built facilities and two activity sessions which means six facility-session pairs. Suppose that customers all live in the same location, say location 1, the preferences of customers and residual capacity of facility-session pairs are listed in Table 1. We assume that the total demand of the customers at location 1 is 1.

Table 1 The Preference and Residual Capacity of the Example

(j, t)	$p_{1,j,t}$	residual capacity
(1, 1)	0.1	1
(1, 2)	0.2	1
(2, 1)	0.3	0.3
(2, 2)	0.4	0.3
(3, 1)	-0.5	0.5
(3, 2)	-0.6	0.5

In this case, no one will go to facility 3 in any activity session since the customers hold negative preference over pairs (3, 1) and (3, 2), which is lower than the zero utility of staying at home. Therefore, facility 3 does not need to be taken into consideration. If the customers choose facility-session pairs (1, 2) and (2, 2) with proportion 0.7 and 0.3, the status is presented in Table 2. Since only pair (2, 2) is fully occupied (i.e. $w_{2,2}=0$), according to constraint (11), the preference of the most preferred available pair among available ones is $p_{1,2,1}$ (i.e. $z_1=0.3$). Constraint (12) restricts $\bar{x}_{1,1,2}$ to be 0 since it is not the most preferred one. However, this creates a contradiction with the result given by constraint (13), where $\bar{x}_{1,1,2}=1$. The above discussion is summarized in Table 2.

Table 2 The Case Violating Preference Constraint

(j, t)	$p_{1,j,t}$	$x_{1,j,t}$	residual capacity	w_{jt}	$\bar{x}_{1,j,t}$ in (12)	$\bar{x}_{1,j,t}$ in (13)
(1, 1)	0.1	0	1	1	0	0
(1, 2)	0.2	0.7	0.3	1	0	1
(2, 1)	0.3	0	0.3	1	1	1
(2, 2)	0.4	0.3	1	0	1	1

The above solution ($x_{1,1,2}$=$x_{2,2,2}$>0) cannot be a valid equilibrium outcome is due to the fact that there exists an available facility-session pair that is more preferred than one that is chosen by some customers (i.e., $p_{1,2,1}$>$p_{1,1,2}$ while (2, 1) still has residual capacity). For this example, the only valid equilibrium that satisfies all preference constraints is listed in Table 3. In this case, the customers choose to go to facility 1 in session 2 and facility 2 in both sessions with proportion 0.4, 0.3, and 0.3, respectively. Similarly, according to constraint (11), the value of z_1 is 0.2, and no constraint is violated.

In short, our formulation guarantees that if there is still any available facility that a customer prefers more, the customer will go to the more preferred one instead of others. Therefore, each customer will act to maximize her/his preference.

Table 3 The Case Satisfying Preference Constraint

(j, t)	$p_{1,j,t}$	$x_{1,j,t}$	residual capacity	w_{jt}	$\bar{x}_{1,j,t}$ in (12)	$\bar{x}_{1,j,t}$ in (13)
(1, 1)	0.1	0	1	1	0	0
(1, 2)	0.2	0.4	0.6	1	1	1
(2, 1)	0.3	0.3	0	0	1	1
(2, 2)	0.4	0.3	0	0	1	1

3.4 A Note on the Activity Sessions

While the sets of customers I and facilities J are pretty much given, the set of activity sessions T is artificially determined by the decision maker. One may wonder how a practitioner may determine the time unit and number of activity sessions T in a time unit when applying this model. We briefly discuss this issue in this section to provide a guide for practitioners.

One basic rule is that a time unit should be chosen so that a customer rarely wants to visit a facility more than once in a day. For example, if customers are residents and facilities are parks, courts, and sporting centers for residents to exercise there, one day should be a good candidate of a time period. The length of an activity session should then be determined with the following consideration. First, a customer's preference of visiting a facility should be roughly the same within the same activity session. It is thus not a good idea to split one day into only two activity sessions "midnight to noon" and "noon to midnight", because one's preference to jog on streets around 7 AM and 11 AM should be significantly different. Second, the length of an activity session should also be long enough so that it is reasonable for one to distinguish two different activity sessions. For example, splitting a day into 24 or even 48 activity sessions when consideing customers' visiting to sport facilities can be a bad idea.

In short, the length of an activity session should be chosen so that the estimation of customers can be reasonably performed, and the number of activity sessions may then be determined accordingly.

4. Algorithms and Analysis

In this section, we propose an iterative greedy heuristic algorithm for our problem.

4.1 Greedy Selection Algorithm with Maximum Flow (GSAMF)

4.1.1 The Algorithm

We propose a greedy algorithm that in each iteration, we select the facility and its scale levels with the best performance ratio among those unbuilt ones and add it into the construction plan. The selection process stops when there is not enough budget to build more new facilities.

As described in Section 3, among the built facilities, the customers will choose where to go according to their preference order. Therefore, once we are given a set of built facilities with determined scale levels, represented by $y=[y_{jk}]_{j\in J,\, k\in K}$, we have to find the number of customers served by this plan, i.e., to solve the objective value $z(y)$ of the following program

$$z(y) = \max \sum_{i\in I} \sum_{j\in J'} \sum_{t\in T} d_i x_{ijt} ,$$

$$\text{s.t.} \quad (4), (9) - (16).$$

Kang et al. (2023) proposes a way to transform the benefit evaluation problem to a maximum flow problem. We now show how to extend this method to incorporate time dependency, which is not considered in his work. Given a construction plan y, we construct a directed acyclic graph whose structure is similar to that in Figure 1. For each customer location i, we add a customer node C_i into the graph. A source node S is created and connected to customer node C_i with capacity d_i for all $i \in I$. For facility j, if under the given construction plan it is built at scale level k (i.e., y_{jk}=1), we add a node F_{jt} for each of activity session t. A destination node D is created and linked from node F_{jt} with capacity q_{jk}. Finally, a link from node C_i to node F_{jt} is added with infinite capacity if $p_{ijt} > 0$. Figure 1 is an example for three customers, two activity sessions, and a construction plan that builds three facilities. According to the way we connect C_i and F_{jt}, we find customer 1 is unwilling to visit facility 2 in activity session 1. Once the graph is constructed, its maximum flow may be solved. For the maximum flow instance made from a construction plan y, we denote the maximized flow value as $w(y)$.

Note that the information about preference levels p_{ijt} is largely omitted in the constructed maximum flow problem: It only matters whether $p_{ijt} > 0$ or not. It is thus not surprising that the solutions to the two problems are not the same. Nevertheless, as for the benefit evaluation problem all we need is the objective value, the constructed maximum flow problem is enough. We now show in Theorem 1 that the maximized flow value of the maximum flow problem constructed given a construction plan y is always identical to $z(y)$, the objective value of the benefit evaluation problem.

Theorem 1: For any construction plan y, we have $z(y)=w(y)$.

Proof. Given any construction plan y, each flow on the graph we constructed is bounded by the demand of customers and capacity of facilities. Therefore, the capacity constraints in benefit evaluation problem are obeyed. For every node C_i, if the maximum flow flows out according to the preference levels in equilibrium, the customers' decisions in graph are exactly the same as that in the benefit evaluation problem. It turns out $z(y)=w(y)$. Based on this maximum flow instance, assume that there is at least one flow flowing out some node C_i changes to the less preferred edge. If this change does not decrease $w(y)$, i.e., does not make any edge out of capacity, $z(y)=w(y)$. If it does, this new instance is impossible to be an outcome of the maximum flow problem. In conclusion,

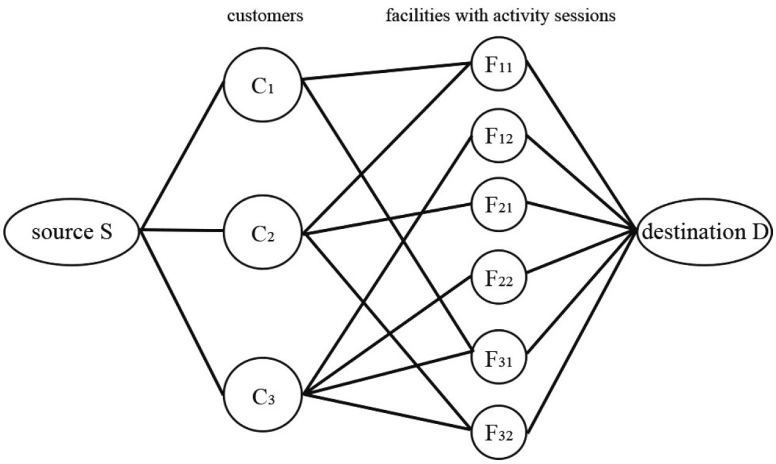

Figure 1 An Example of the Maximum Flow Graph

even we allow the customers to choose the facility which is not her/his most preferred in constructed maximum flow, $z(y)=w(y)$ still holds.

Theorem 1 provides us a way to solve the benefit evaluation problem. With this, we are now ready to describe the structure of our proposed algorithm, GSAMF. To do this, we further define two functions $f(y)$ and $Ratio(j,k|y)$. The function $f(y)$ is the total fixed construction cost given a construction plan y, i.e.,

$$f(y)=\sum_{j\in J}\sum_{k\in K} f_{jk}y_{jk}.$$

The function $Ratio(j,k|y)$ is the benefit-to-cost ratio by adding facility j with scale level k into a construction plan to form a new plan y. Let $y^{\text{origin}}(j_0,k_0|y)$ be the original construction plan before adding facility j_0 at scale level k_0 to form plan y, i.e.,

$$y_{jk}^{\text{origin}}(j_0,k_0|y)=\begin{cases} y_{jk}-1, & \text{if } (j,k)=(j_0,k_0) \\ y_{jk}, & \text{otherwise} \end{cases} \quad \forall j\in J, k\in K.$$

The ratio function is then defined as[3]

$$Ratio(j,k|y)=\frac{\text{marginal objective value}}{\text{marginal construction cost}}=\frac{z(y)-z\left(y^{\text{origin}}(j,k|y)\right)}{f_{jk}}.$$

3 We have examined the performance of four different ratios, where the numerator may be $z(y)$ or $z(y)-z\left(y^{\text{origin}}(j,k|y)\right)$, and the denominator may be $f(y)$ or f_{jk}. It turns out that the combination of $z(y)-z\left(y^{\text{origin}}(j,k|y)\right)$ and f_{jk} results in the best performance.

In each iteration of GSAMF, we are given an original construction plan. We test each unbuilt facility with each candidate scale level by calculating the benefit-to-cost ratio upon adding it into the given construction plan. After choosing the one resulting in the highest ratio, we proceed to the next iteration. We stop until all locations have been chosen or we run out of budget. The pseudocode of the greedy selection algorithm with maximum flow is presented in Algorithm 1.

Algorithm 1 Greedy Selection Algorithm with Maximum Flow (GSAMF)

```
y ← 0,S ← ∅
repeat
bestRatio ← 0,(j*,k*) ← (0, 0)
  for j ∈ J \ S do
    for k ∈ K do
      if f(y) + f_jk ≤ B then
        y_jk ←1
        aRatio ← Ratio(j,k|y)
        if aRatio > bestRatio then
          bestRatio ← aRatio,(j*,k*)←(j,k)
        end if
        y_jk ←0
      end if
    end if
  end if
  y_j*k* ←1,S←S∪{ j* }
until (j*,k*)=(0,0)
return y
```

4.1.2 Time Complexity

Let V and E be the sets of the nodes and edges in the graph, the numbers of V and E satisfy $|V|\leq|I|+|J||T|+2$ and $|E|\leq|I||J||T|+|I|+|J||T|$. A classic way to solve maximum flow problem is to use the Edmonds-Karp algorithm proposed in Edmonds and Karp (1972), which is an implementation of Ford-Fulkerson method using breadth-first search in finding the augmenting path. The Edmonds-Karp algorithm provides a solution with a $O(|V||E|^2)$ bound. Therefore, in our problem, the time complexity of solving the maximum flow problem is $O(|V|\ |E|^2)=O(|I|^3\ |J|^2|T|^2)$.

In each iteration, our algorithm spends $O(|I|^2 |J|^3|T|^3)$ to compute the number of the customers served by the construction plan. The algorithm runs for at most $|J|$ iterations. In the j^{th} iteration, it solves at most $(|J|-j+1)|K|$ maximum flow problems, each with j facility nodes in the graph, which means the complexity of completing the j^{th} iteration is $O(|I|^2 J^3|T|^3)$. Therefore, the total time complexity is

$$O\left(\sum_{j=1}^{|J|}(|J|-j+1)\,|K|(|I|^2 j^3 |T|^3)\right) = O(|I|^2|J|^5|T|^3|K|).$$

4.1.3 Incremental Maximum Flow

In each iteration of GSAMF, we try all unbuilt facilities to add one into the construction plan with solving the maximum flow problem, which takes long computational time. However, in our algorithm, there are only slight changes of the graph comparing to those in previous iterations; most of the flows are still the same. We only need to find new augmenting path and the backward path after adding new facility nodes and edges. More precisely, adding a new edge is equivalent to changing one edge's capacity from zero to some positive number on the previous solved graph. The difference between previous maximum flow and the new one will only be determined by those vertices affected by this insertion. Therefore, whenever we add a new facility into the construction plan, instead of creating a whole new graph, we add vertices and edges to the previous solved graph and then continue solving the maximum flow problem.

Kumar and Gupta (2003) propose an incremental algorithm for the maximum flow problem of inserting an edge in the graph. The time complexity of the algorithm is $O(|\Delta V|^2|E|)$, where $|\Delta V|$ is the number of affected vertices and $|E|$ is the number of edges. When an edge is inserted into the graph, there may exist new augmenting path from source to sink through the new edge. The affected vertices are those that lie on at least one augmenting path. That is, when we add a new facility j into the construction plan, it is equivalent to insert $|T|$ edges, i.e., (f_{jt}, d) for all $t \in T$, to the graph and each insertion affects $|\Delta V|=O(|I|+3)$ nodes. Therefore, the total time complexity of adding a new facility to the graph is

$$\mathrm{O}(|\Delta V|^2|E||T|) = O\Big(\big((|I|+3)^2(|I|+|I||J||T|+|J||T|)\big)|T|\Big) = O(|I|^3|J||T|^2),$$

and the total time complexity is

$$O\left(\sum_{j=1}^{|J|}(|J|-j+1)\,|K|(|I|^3 j|T|^2)\right)=O(|I|^3|J|^3|T|^2|K|).$$

The implementation with incremental maximum flow indeed reduces the time complexity of GSAMF.

4.2 Greedy Selection Algorithm with Maximum Flow Estimation (GSAMFE)

In our algorithm, given a construction plan y, we have to calculate the corresponding number of served customers. However, considering the large number of customers and facilities, it requires long computational time even using the incremental maximum flow idea may still be too time-consuming. Therefore, we adopt the flow estimation algorithm proposed in Kang et al. (2023) to evaluate the objective value and extend it to the scenario with time dependency.

Let I_{jt} be the set of customers that $p_{ijt} > 0$ for facility j at activity session t and I_{jt} be the set of facilities that $p_{ijt} > 0$ for customer i at activity session t. Given any construction plan y, we define two variables $\alpha_i(y)$ and β_{jk}, where the former is called the potential demand of customer i, and the latter is called the potential supply of facility j of scale level k. More precisely, we define

$$\alpha_i(y)=\min\left\{d_i,\sum_{t\in T}\sum_{j\in J_{it}}\sum_{k\in K}q_{jk}y_{jk}\right\}$$

and

$$\beta_{jk}=\sum_{t\in T}\min\left\{\sum_{i\in I_{jt}}d_i,q_{jk}\right\}.$$

The estimated objective value $z'(y)$ of the construction plan y is the minimum of the potential demand and supply, i.e.,

$$z'(y)=\min\left\{\sum_{i\in I}\alpha_i(y),\sum_{j\in J}\sum_{k\in K}\beta_{jk}\right\}.$$

In our algorithm with flow estimation, we replace $z(y)$ in GSAMF with its estimated value $z'(y)$. We therefore replace $Ratio(j,k|y)$ by $Ratio'(j,k|y)$, which is defined as

$$Ratio'(j,k|y) = \frac{z'(y) - z'\left(y^{\text{origin}}(j,k|y)\right)}{f_{jk}}.$$

The pseudocode of the greedy selection algorithm with maximum flow estimation (GSAMFE) is presented in Algorithm 2. The only difference between Algorithms 1 and 2 is using $Ratio(j,k|y)$ or $Ratio'(j,k|y)$ (i.e. $z(y)$ or $z'(y)$) in each iteration. While this is some kind of approximation, a huge amount of computation time may be saved.

We now analyze the complexity of GSAMFE. First of all, $\sum_{j\in J}\sum_{k\in K}\beta_{jk}$ should be calculated before the greedy selection starts. The time it takes is $O(|I||J||K||T|)$. Second, the values of $\alpha_i(y)$ should be initialized to d_i for all customer i. This takes $O(|I|)$. The final part is the greedy selection, whose structure of GSAMFE is the same as that of GSAMF. Therefore, GSAMFE also runs for at most $|J|$ iterations, and in the j^{th} iteration, it does at most $(|J|-j+1)|K|$ times of flow estimation. For each time of flow estimation, a new facility of a certain scale level is added, which requires GSAMFE to update the values of $\alpha_i(y)$ for all customers who are willing to visit the newly added facility in at least one activity session. Such updating can be done in $O(|I||T|)$. Collectively, the total time complexity is

$$O\left(|I||J||K||T| + |I| + \sum_{j=1}^{|J|}(|J| - j + 1)|K|(|I||T|)\right) = O(|I||J|^2|K||T|).$$

Compared to GSAMF, which solves for the exact flow amount for each construction plan, GSAMFE indeed saves time with the idea of flow estimation.

4.3 An Illustrative Example

In this section, we demonstrate an example of the GSAMFE algorithm to better explain how it works. Suppose that there are three customers, three facilities, and two activity sessions. For each facility, there are two scale levels. The budget of the construction plan B=70, and the customer demands, facility capacities, and facility construction costs are listed in Tables 4 and 5. In Table 6, we label the preference levels larger than 0 with the plus sign (+).

Algorithm 2 Greedy Selection Algorithm with Maximum Flow Estimation (GSAMFE)

```
y ← 0,S ← ∅
repeat
bestRatio ← 0,(j*,k*) ← (0, 0)
  for j ∈ J \ S do
    for k ∈ K do
      if f(y) + f_jk ≤ B then
        y_jk←1
        aRatio ← Ratio'(j,k|y)
        if aRatio > bestRatio then
          bestRatio ← aRatio,(j*,k*)←(j,k)
        end if
        y_jk←0
      end if
    end if
  end if
  y_j*k* ←1,S←S∪{ j* }
until (j*,k*)=(0,0)
return y
```

Table 4 Example Demands

customer i	demand d_i
1	4
2	9
3	11

Table 5 Example Capacities and Construction Costs

facility j	scale level k	capacity q_{jk}	construction cost f_{jk}
1	1	4	10
1	2	8	70
2	1	4	35
2	2	8	50
3	1	5	10
3	2	10	60

The preference of customers can be transformed into the graph as a maximum flow problem as Figure 1 in Section 4.1.1.

Table 6 Example Preferences (Positive Ones Are Marked)

customer i	(j, t): facility j with activity session t					
	(1, 1)	(1, 2)	(2, 1)	(2, 2)	(3, 1)	(3, 2)
1	+	+				
2		+			+	
3				+		+

We now start to run GSAMF and GSAMFE on this example. In iteration 1, the estimation step of first iteration, since none of the facilities are added into the construction plan, we compute the estimated flow $z'(y)$ of all facilities with its scale level. The estimated flow $z'(y)$ and the ratio $Ratio'(j,k|y)$ are listed in Table 7. We also list the exact objective value $z(y)$ of our GSAMF algorithm, which solves exact objective value $z(y)$ with maximum flow in every iteration, in the table.

Take facility 3 with scale level 1 for example, if we add it into our construction plan, the customers who are willing to go to those built facilities are customer 2 and 3. Thus, the potential customer demand of each customer is $\alpha_2=\min\{d_2,q_{3,1}\}=\min\{9,5\}$ and $\alpha_3= \min\{d_3,q_{3,2}\} = \min\{11,5\}$, respectively. The potential facility supply of the only built facility 3, β_3, is $\min\{d_2,q_{3,1}\}+\min\{d_3,q_{3,2}\}=\min\{9,5\}+\min\{11,5\}=10$. Therefore, the estimated flow of adding facility 3 with scale level 1 is $z'(y)=\min\{\sum_{i\in I}\alpha_i,\sum_{j\in J}\beta_j\}=\min\{5+5,10\}=10$, and $Ratio'(3,1|y)$ is 1.

According to Table 7, we choose facility 3 with scale level 1 since its $Ratio'(j,k|y)$ is the highest one. After adding it into the construction plan (which is constructing nothing at the beginning of iteration 1), we end this iteration and proceed to the second iteration with only facilities 1 and 2 remain as candidates. The result is shown in Table 8. Note that facility 1 with capacity 2 does not have to take into calculation since we do not have enough budget to build it. At the end of the second iteration, we add facility 1 with scale level 1 into our construction plan. The result is shown in Table 9.

Table 7 Estimated Flows and Ratios in Iteration 1 of the Example

facility *j*	scale level *k*	Budget	$z(y)=z'(y)$	$Ratio(j,k\|y)=Ratio'(j,k\|y)$
1	1	Enough	8	$\frac{4}{5}$
1	2	Enough	15	$\frac{3}{14}$
2	1	Enough	4	$\frac{4}{35}$
2	2	Enough	8	$\frac{4}{25}$
3	1	Enough	10	1
3	2	Enough	19	$\frac{19}{60}$

Table 8 Estimated Flows and Ratios in Iteration 2 of the Example

facility *j*	scale level *k*	Budget	$z(y)=z'(y)$	$Ratio(j,k\|y)=Ratio'(j,k\|y)$
1	1	Enough	18	$\frac{4}{5}$
1	2	Not	–	–
2	1	Enough	14	$\frac{4}{35}$
2	2	Enough	16	$\frac{3}{25}$

Table 9 Estimated Flows and Ratios in Iteration 3 of the Example

facility *j*	scale level *k*	Budget	$z(y)$	$z'(y)$	$Ratio(j,k\|y)$	$Ratio'(j,k\|y)$
2	1	Enough	22	25	$\frac{4}{35}$	$\frac{1}{5}$
2	2	Enough	24	27	$\frac{3}{25}$	$\frac{9}{50}$

At the end of the third iteration, for using GSAMFE algorithm, we add facility 2 with scale level 1 into our construction plan since $Ratio'(2,2|y)$ is the highest one. However, we find that if we apply GSAMF algorithm, we will choose facility 2 with scale level 2 instead of scale level 1 since we use $Ratio(j,k|y)$ instead of $Ratio'(j,k|y)$.

After the third iteration, there is no other location can build more facilities at, so both GSAMFE and GSAMF end iterations. Therefore, for GSAMFE algorithm, the final construction plan y is provided and it will build facility 1, facility 2 and facility 3, all with scale level 1. To get the exact objective value of our outcome, we solve maximum flow with such y. Finally, our GSAMFE algorithm ends with the estimated flow value $z'(y)$ is 25 and the actual objective value $z(y)$ is 22. Note that in this example, if we use our GSAMF algorithm, the final construction plan y will build facility 1 with scale level 1, facility 2 with scale level 2 and facility 3 with scale level 1. It gives objective value $z(y)$=24.

If we solve this example with mixed integer program through solver, we can find that the optimal solution is 24 by constructing facility 1 with scale level 1, facility 2 with scale level 2 and facility 3 with scale level 1. In this example, we find that GSAMFE sometimes may overestimate and choose a solution worse than GSAMF.

5. Numerical Study

5.1 Experiment Setting

To present the experimental results of the algorithm, we adopt seven factors to analyze the performance under different circumstances. The first factor is the size of the problem, which three sizes—small, medium and large—are considered. We set m=10, n=20 as the small size, m=30, n=60 as the medium size and m=100, n=200 as the large size for the problem. The second factor is the number of scale levels which each facility has. We consider two scenarios: one is each facility has one scale level and the other is each facility has three scale levels. The third factor is the numbers of activity sessions which is under two scenarios: problems with only one activity session and with three activity sessions.

The fourth and fifth factors are location preference and time preference which affect the customer's time-dependent preference over facilities. In our experiment, the customer and facility locations are mapped onto a 2-dimentional coordinate. There are two types of customers' location distributions in our setting: uniformly distribution and clustering distribution. In the former type, the locations of customers are randomly separated in the map; in the latter type, customers tend to locate near several clusters. The parameter d_{ij} represents the Euclidian distance between customer i and facility j, and we set customer

i's location preference to facility j to be $\frac{1}{d_{ij}^2}$. As for the time preference, there are two scenarios. One is customers' time preference are homogeneous, i.e., in each activity session, all customers' time preferences are the same. The other is each customer's time preference is randomly distributed. We denote a parameter r_{it} as the time preference of customer i in activity session t. r_{it} is distributed from 0.5 to 1.5. Put these two factors together, the preference of customer i to facility j in activity session t, p_{ijt} is the product of location preference and time preference, i.e., $\frac{1}{d_{ij}^2} \times r_{it}$.

The last two factors are the capacity of facility and the budget of the construction plan, both of which have two types in the experiment. The low-capacity type sets the total supply of facilities to 40 percent of the total demand, while the high-capacity type sets it to 80 percent. The loose budget is set to 40 percent of the total construction cost and the tight budget is set to 80 percent.

The above seven factors generate 3×2×2×2×2×2×2=192 scenarios, and we generate 30 instances for each scenario. The experiments is performed on a PC with a 3.2 GHz Intel(R) Core i7-5820K processor and 16 GB RAM. The heuristic algorithm is implemented in Spyder 4.0 using python 3.6. And the MIP model is solved using 41 Gurobi 8.1 and implemented through Gurobi python.

5.2 Benchmark Algorithm

In order to demonstrate the performance of our algorithm, we implement a genetic algorithm for comparison (see below). First, we randomly generate 100 feasible construction plans into a pool.[4] In each iteration, using the tournament selection method (Miller and Goldberg, 1995), we randomly select 5 plans from the pool and pick the best two among them. Then we implement crossover to those two plans. We randomly pick a cross-point and divide both plans into head and tail. Then we switch one's head with other's head to form two new plans. The two new solutions have 10% chance to mutate. If a solution mutates, it will randomly pick one unbuilt facility with one scale level and add it into the construction plan with the budget. Finally, two new solutions will be added

4 It may be wondered whether 100 is a parameter value for the implementation of the genetic algorithm. Therefore, we investigate this issue by examining the performance of the genetic algorithm with other parameter values at the end of Section 5.

into the pool if they are feasible and the worst two plans will be removed from the pool. Besides, for the small and medium sizes of problem, we run the genetic algorithm for 2,000 iterations; for the large size problem, the number of iterations is 10,000. After iterations, the genetic algorithm will return the best construction plan in the pool.

5.3 Solution Performance

In this section, we use z to denote the objective value of the solution found by GSAMFE, and z^* to denote that found by mathematical model.

In Table 10, we find that our algorithm performs well in three problem sizes and average performance is better in the medium size. When the problem size increases, there is more chance to select non-optimal facilities. However, the effect of selecting single wrong facility and scale level may also impact less on the total objective value when the problem size increases. Moreover, the number of iterations increase with the problem size. Therefore, even if the algorithms choose a non-optimal facility in an iteration, there are more chances for them to choose good facilities in the following iterations.

Table 11 shows that our algorithm performs well in two scenarios, and it performs better when there is only one scale level. If a facility has multiple scale levels, the difficulty of the problem increases. There are more chances to pick the non-optimal facility and scale levels, so our algorithm is easily to get a better solution when the number of scale levels is less.

Table 10 Numerical Result of Problem Size

Instance size	Average		Minimum	
	$\frac{z}{z^*}$	$\frac{z^{GA}}{z^*}$	$\frac{z}{z^*}$	$\frac{z^{GA}}{z^*}$
Small	0.9855	0.9308	0.7231	0.6702
Medium	0.9915	0.8558	0.8342	0.5917
Large	0.9857	0.7865	0.6656	0.5189

Table 11 Numerical Result of Number of Scale Level

Number of scale levels	Average		Minimum	
	$\frac{z}{z^*}$	$\frac{z^{GA}}{z^*}$	$\frac{z}{z^*}$	$\frac{z^{GA}}{z^*}$
1	0.9904	0.9202	0.6656	0.5964
3	0.9847	0.7952	0.7231	0.5189

Table 12 shows that the performance in three activity sessions is better. When there are more activity sessions, the number of customers' options also increases. Therefore, even if the algorithm chooses non-optimal solutions, there is more chance that some of the customers' demand can still be served by other facilities.

Table 13 shows that the performance is better when the distribution of customer location is cluster. It is because when customers are located clustered, it is likely to serve more customers when building a new facility near the cluster center. As for the time preference distribution, they perform better in random time preference distribution. If customers tend to go to facilities in different activity sessions, there is less chance that facilities are full, so it is easier for algorithms to get good solutions.

Table 12 Numerical Result of Number of Activity Sessions

Number of activity sessions	Average		Minimum	
	$\frac{z}{z^*}$	$\frac{z^{GA}}{z^*}$	$\frac{z}{z^*}$	$\frac{z^{GA}}{z^*}$
1	0.9854	0.8566	0.7231	0.5189
3	0.9897	0.8588	0.6656	0.5848

Table 13 Numerical Result of Customer Location and Preference Distribution

Location and preference distributions	Average		Minimum	
	$\frac{z}{z^*}$	$\frac{z^{GA}}{z^*}$	$\frac{z}{z^*}$	$\frac{z^{GA}}{z^*}$
cluster, cluster	0.9936	0.8568	0.7667	0.5848
cluster, random	0.9936	0.8561	0.8693	0.5755
random, cluster	0.9747	0.8558	0.6656	0.5189
random, random	0.9883	0.8622	0.7231	0.6471

Finally, Table 14 shows that our algorithm performs well no matter under which capacity and budget constraints. Note that this is not true for the genetic algorithm. In other words, the performance of GSAMFE is not prone to the tightness of capacity or budget.

We finish this section by examining the impact of the parameter value of the genetic algorithm. Our goal is to investigate whether 100 is a pool size for the genetic algorithm to have good performance. Thus, we adopt alternative pool sizes 25, 50, and 150 to see whether different pool sizes result in significantly different performance of the genetic algorithm.

Table 14 Numerical Result of Capacity and Budget Constraints

Capacity and budget	Average		Minimum	
	$\frac{z}{z^*}$	$\frac{z^{GA}}{z^*}$	$\frac{z}{z^*}$	$\frac{z^{GA}}{z^*}$
large capacity and large budget	0.9864	0.8826	0.8046	0.6496
large capacity and small budget	0.9809	0.8264	0.7231	0.5189
small capacity and large budget	0.9886	0.8845	0.6656	0.6473
small capacity and small budget	0.9944	0.8374	0.8357	0.5755

The average optimality gaps $\frac{z^{GA}}{z^*}$ are presented in Table 15.[5] As different pool sizes do not result in significantly different performance, we conclude that the results we obtain from Tables 10 to 15 are reasonable.

Table 15 Average Optimality Gap of the Genetic Algorithm

Instance size	Pool size			
	25	50	100	150
Small	0.8764	0.8926	0.9020	0.8956
Medium	0.8338	0.8579	0.8548	0.8392
Large	0.7705	0.7884	0.7786	0.7687

5 By using the same experiment setting to generate random instances to construct Table 15, we conduct a new numerical experiment which is independent of that generating Tables 10 to 14. This is why the fourth column of Table 15 is not completely the same as the third column of Table 10.

6. Conclusion

In this study, we consider a capacitated facility location problem with time-dependent preferences. Inspired by previous literature, we reformulate the problem into a single-level mixed integer problem with the objective to maximize the total number of customers served. Since the problem is NP-hard, we develop two greedy-based heuristic algorithms with maximum flow network and flow estimation, respectively. The latter is significantly more time efficient. Through our numerical study, we find that our second algorithm can provide near-optimal solutions in reasonable much shorter time.

There are several ways to extend this study. In particular, one decision that is missing in our model is for the decision maker to determine the equipment/services to be delivered in each facility. For example, if the decision maker is building sport facilities, it is the job of this decision maker to allocate the limited space to basketball courts, swimming pools, fitness rooms, etc., which certainly will affect potential customers' preference over different facilities. Extending our model and algorithm to include this feature will make them more applicable in practice. Another research direction is to investigate the proposed algorithm from a more theoretical perspective to see whether there is a worst-case performance guarantee. An investigation on this may generate analytical contributions to the literature of discrete optimization.

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Author Biography

*Ling-Chieh Kung

Ling-Chieh Kung is currently an Associate Professor in the Department of Information Management, National Taiwan University. One of his research interests is to apply theoretical economic modeling and analysis to study the design, pricing, and contracting problems of multi-sided platforms and digital goods/services. He has published journal articles in *IEEE Transactions on Automation Science and Engineering*, *European Journal of Operational Research*, *Journal of the Operational Research Society*, *Transportation Research Part E*, *Naval Research Logistics*, *Operations Research Letters*, *Computers and Operations Research*, among others. He obtained his Doctoral degree from the Department of Industrial Engineering and Operations Research, University of California, Berkeley, USA. He obtained his Master's and Bachelor's degrees both from the Department of Information Management, National Taiwan University, Taipei, Taiwan.

Jih-Sheng Chuang

Jih-Sheng Chuang graduated from the Department of Information Management, National Taiwan University with both the Master's and Bachelor's degrees. He completed his master thesis with Professor Ling-Chieh Kung as his thesis adviser.

Yun-Tung Kuo

Yun-Tung Kuo graduated from the Department of Information Management, National Taiwan University with the Master's degree. She obtained her Bachelor's degree from the Department of Physics, National Taiwan University, Taipei, Taiwan. She completed her master thesis with Professor Ling-Chieh Kung as her thesis adviser.

*E-mail: lckung@ntu.edu.tw

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