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Optimal Advertorial Allocation and Contract Design of a Multichannel Networks Company on Video Sharing Platforms

影片分享平臺中多頻道聯播網之最佳業配任務分配與合約設計

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Abstract

Multichannel Networks (MCN) companies arise due to booming video-sharing platforms like YouTube. MCN companies sign creators who produce and publish videos on video-sharing platforms. In addition to the advertising revenue from the platforms, the companies and creators can earn revenue by completing advertorial tasks delegated by business owners. In this study, we study how an MCN company allocates an advertorial task and designs revenue-sharing contracts for creators with different abilities to attract online viewers. We find that the MCN company may not always allocate an advertorial task only to the most outstanding creator. When the creators' abilities are not too distinct, and the advertorial fee and the cost for creating a video are moderate, a splitting strategy (i.e., allocating the task to multiple creators) could be optimal. By comparing different industry structures, we also show that the splitting strategy cannot be optimal without independent MCN companies. In other words, independent MCN companies may make video-sharing platforms more diversified.

【Keywords】online video sharing, multichannel networks, advertorials, revenue sharing, game theory

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摘 要

多頻道聯播網公司 (MCN) 隨著 YouTube 等影片分享平臺的蓬勃發展，在數位世代興起一股風潮。MCN 與在影片分享平臺發布作品的創作者簽約，除了和創作者共同賺取來自平台的廣告收入外，也通過完成企業主委託的業配任務來賺取收益。本研究透過賽局分析建模，探討當 MCN 簽下多個吸金能力不同的創作者時，應該如何分配業配任務並設計與創作者簽訂的營收分享合約。此外，本研究也探討 MCN 的存在會如何影響行業環境。我們發現 MCN 不該總是將業配任務分給能力最強的創作者。當創作者的能力彼此相差不遠、業配費用以及製作影片的成本適中時，拆分策略（亦即將業配任務分配給不同創作者）可能是最佳的。另外，通過比較不同的行業結構，我們也發現若是沒有獨立的 MCN 公司，拆分策略就不會是最佳策略。換言之，獨立 MCN 的存在為能力較弱的創作者創造機會，使影片分享平臺更加多元化。

【關鍵字】影片分享平臺、多頻道聯播網、業配廣告、營收分享、賽局理論

1. Introduction

Founded in 2005, YouTube has become one of the most dominant online video-sharing platforms in the world. It is reported that there are two billion logged-in viewers every month on the platform, which amount to almost one-third of Internet users (Spangler, 2019). YouTube users are allowed to upload user-generated content onto the platform. As of 2022, more than 51 million channels exist on YouTube, and more than 500 hours of video content are uploaded to YouTube every minute (Funk, 2022; Hale, 2019).

Revenue sharing of video advertising, automated content ID, and open viewer metrics comprise a massive YouTube commercial ecosystem (Lobato, 2016). As a result, creating content on YouTube has become an occupation recently. A person who invests a lot of time producing videos on YouTube is commonly titled a YouTuber (Holmbom, 2015). According to Forbes, the highest-paid YouTuber in 2020 is Ryan Kaji, who earned 29.5 million U.S. dollars before management fees and taxes (Berg and Brown, 2020). For more general expression, we use the term “creator” to refer to “YouTuber” or any similar people on other video-sharing platforms throughout this article.

While creators typically focus on video production, they may lack the experience and know-how to do business. Multichannel Networks, also known as MCNs, thus arise as intermediary firms that connect entrepreneurial creators with advertising, marketing, and screen production industries (Lobato, 2016). MCNs may also help creators increase audience and advertising income. In general, the services provided by MCNs include managing creators’ commercial relationships, cross-promoting affiliated channels, providing technical services such as managing intellectual property, etc. As an exchange, MCNs share a proportion of revenue (typically 20-50% of net advertising revenues) as a commission (Lobato, 2016). Fullscreen, one of the largest MCNs nowadays, contracts with about 2,500 content creators who together generate seven billion views per month (Weiss, 2020).

Since MCNs and creators split the revenue, they have the same revenue sources. The major sources of income are advertising revenue paid by YouTube and sponsored activities, such as advertorial fees.[1] When people watch videos on YouTube,

1 “Advertorial” is an advertisement that imitates editorial format. Advertorial Definition, MERRIAM-WEBSTER, http://www.merriam-webster.com/dictionary/advertorial (last visited Jan. 7, 2021).

advertisements are displayed before, after, or in the middle of the videos. YouTube shares the advertising revenue with creators (Miller, 2010). Besides, creators may also create videos to specifically market a brand or a product in exchange for advertorial fees (Wu, 2016). According to Stephanie Horbaczewski, the founder of the fashion MCN Style Haul, sponsored activity is a more valuable revenue source than advertising revenue sharing since the former is non-Google dependent.[2] Moreover, since sponsored activities often express deeper brand messages and secure higher consumer eyeballs, they are more lucrative (Lobato, 2016). There are many forms of sponsored activities, among which making advertorial content is the most common one on YouTube. Therefore, we focus on advertorial activities throughout the study.

As the intermediaries between creators and business owners (sponsors), it is challenging for MCNs to set the revenue sharing percentage and allocate the opportunities for advertorials to maximize profit. First, MCNs, creators, and business owners all care about their own profit, thus having misaligned profit incentives. Second, the revenue sharing percentage and the advertorial allocation decision are intertwined and should not be determined separately. Third, creators have different popularity and may exert different effort levels. An MCN must estimate how its decisions may affect its creators' effort decisions, making it even more difficult to determine who to take charge of the advertorial. These make an MCN's contract design and advertorial allocation problem challenging.

In this study, we would like to investigate the following questions: (1) How should MCNs allocate the advertorial? (2) How should MCNs set the revenue sharing proportions with the creators? (3) Who benefits from the existence of MCNs? (4) How the industry structure affects the equilibrium decisions? With the aim of tackling these problems, we build three game-theoretic models considering three structures of interaction among an MCN and multiple effort-exerting creators with different abilities. The major purpose of our research is to study the profitability of feasible advertorial allocation strategies, figure out factors that affect MCN's equilibrium choice, and compare the differences among different industry structures. Our findings may provide managerial implications for both MCNs and creators.

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2 The advertising revenue sharing program, called Google AdSense, is run by Google, the parent company of YouTube.

2. Literature Review

In recent years, issues related to information goods have been widely concerned (Aral and Dhillon, 2021; Chellappa and Mehra, 2018; Dou, Hu, and Wu, 2017; Liu, Hung, and Hsiao, 2019; Zhang, Nan, and Tan, 2021). For digital goods, making online advertorials is one of the most important income sources. While some previous works investigate the word-of-mouth effect (see, e.g., Abubakar and Ilkan, 2016; Litvin, Goldsmith, and Pan, 2008), some discuss how the content of an advertorial should be designed. It is documented that consumers are typically defensive when viewing an advertorial (Darke and Ritchie, 2007). Therefore, some researchers suggest creators truthfully provide negative comments when producing an advertorial (see, e.g., Hwang and Jeong, 2016). However, the contracting issue, which is the main focus of our work, is missing in this stream of literature.

Our paper also relates to revenue sharing for the production of digital goods/services. Sun and Zhu (2013) examine how the launch of an ad-revenue-sharing program would affect the behaviors of bloggers. Their results show that with such a revenue-sharing system, bloggers tend to provide higher-quality content and devote themselves to more popular topics. Bhargava (2021) develops an analytical model in which a platform aggregates plenty of content created by multiple producers into a bundle and shares its revenue with the producers. He investigates demand, production choices, and revenue-sharing arrangements in a cross-producer bundle economy. Jain and Qian (2021) analytically study how a revenue-sharing contract between a monopoly digital content platform and several independent producers would be affected by multiple factors, including the nature of competition among various producers, the size of customer base, and the type of customers. These studies indicate that revenue sharing is a popular contract format for digital content creation. We add to this stream of literature by investigating the revenue-sharing relationship between an MCN company and creators on video-sharing platforms.

The contracting issue between an MCN company and a creator is similar to the traditional salesforce compensation problem (Lal and Staelin, 1986; Rao, 1990). Contract design with respect to an agent who privately exerts costly effort has been studied in the context of channel coordination (Taylor, 2002), threshold incentives (Sohoni, Chopra, Mohan, and Sendil, 2011), and demand forecasting (Chen, 2005; Kung and Chen, 2014). Nevertheless, to the best of our knowledge, there is no work dedicated to the contract design problem of an MCN company on a video-sharing platform with the consideration of advertorial allocation. Our work thus contributes to the literature.

3. Model

To answer our research questions, we construct stylized models to describe the interaction among an independent MCN company and multiple creators. To better understand the impact of the existence of the MCN company, we consider three types of industry structures: (1) the independent-MCN structure (structure I), (2) the leagued-MCN structure (structure L), and (3) the no-MCN structure (structure N). These three types of models are based on different kinds of cooperative relationships among creators and MCN companies in the real business world. Structure I reflects the most common format adopted by famous MCN companies, including Fullscreen (based in America) and CAPSULE (based in Taiwan), and many other creators; structure L is built up based on studios established by a group of creators who are close friends or siblings (such as This Group Of People and Huang Brothers in Taiwan); structure N depicts the scenario which most of the beginning YouTubers face at their initial stage. By characterizing the players' optimal decisions and equilibrium outcomes, we draw managerial implications.

3.1 The Independent-MCN Structure (I)

3.1.1 The Basic Setting

Under the independent-MCN structure, we have three players: an MCN company (it), a high-type creator (she), and a low-type creator (he), where the high-type creator is more capable of attracting audiences than the low-type one (e.g., by having a larger number of subscribers or more views per video). Each of the two creators may make a video. The industry structure with an independent MCN is depicted in Figure 1.

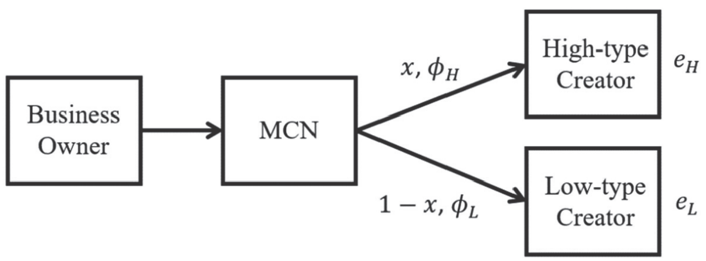

Figure 1 The Independent-MCN Structure

There are two revenue sources for a video: (1) the advertisement revenue paid by a video sharing platform (e.g., YouTube) and (2) the advertorial fee paid by a business owner. The MCN company, delegated the advertorial task from a business owner, must determine two things: (1) how to set the revenue sharing proportions with the two creators respectively, and (2) how to allocate the advertorial fee to the two creators. The revenue percentages left for the high-type and low-type creators are ϕ_H and ϕ_L, respectively, where $i \in \{H,L\}$; while the advertorial share given to the high-type creator is x and 1-x is given to the low-type, where $x \in [0,1]$. These two decisions both affect the creators' decisions on exerting costly efforts. For the type-i creator's effort level to be e_i, she/he pays $\frac{k}{2}e_i^2$ as the cost, where $k > 0$ is an exogenous parameter.

The amount of advertisement revenue earned from the sharing platform is determined by the number of views and the per-view payment γ provided by the sharing platform. The number of views is determined by three things: (1) the natural traffic a, which is determined by the number of the sharing platform's users, (2) the effort level e_i that type-i creator puts on the video, and (3) the ability β_i of the type-i creator to attract audiences, where $\beta_H > \beta_L$. Increasing any of these three items increases the expected number of views. Collectively, the total amount of advertising revenue paid by the sharing platform for a video is $(a+e_i)\beta_i\gamma$.[3]

Advertorial fee A from business owners works differently. To pay for an advertorial, a business owner would request a certain performance threshold (e.g., views or clicks), which may or may not be met. The probability for a video to meet the threshold is determined by both the creator's effort level e_i and ability β_i. The higher the effort level or ability, the higher the probability of meeting the threshold. Moreover, the effort level and ability are complementary: exerting effort is more effective for a creator with higher ability. Thus, the probability of meeting the threshold for type-i creator is $\beta_i e_i$, while the failed one is 1-$\beta_i e_i$. The business owner pays the advertorial fee if and only if the threshold is met by at least one creator or nothing otherwise.[4] The three scenarios with at least one creator meeting the threshold, their probabilities of occurrence, and the advertorial

3 The additive format of the effect of effort exertion and the convex setting of its cost have been widely adopted in literature, including Lal and Staelin (1986), Chen (2005), and Zhu and He (2017), among others.

4 In practice, a creator may obtain a certain amount of compensation based on the performance of the video. To avoid tedious analysis, we omit this possibility.

Table 1 Expected Advertorial Revenue For MCN

Who meet(s) the threshold	Probability	Advertorial revenue
Both creators	$\beta_H e_H \beta_L e_L$	$(1-\phi_H)xA+(1-\phi_L)(1-x)A$ *
High-type creator	$\beta_H e_H (1-\beta_L e_L)$	$(1-\phi_H)xA+(1-x)A$
Low-type creator	$(1-\beta_H e_H)\beta_L e_L$	$xA+(1-\phi_L)(1-x)A$

Note:* The remaining $\phi_H\, xA$ goes to the high-type creator, whereas the remaining $\phi_L\, (1\text{-}x)A$ goes to the low-type creator. The revenue-sharing contract is modelled following the most classic setting in literature. Interested readers may see Cachon and Lariviere (2005) for more information.

revenues for the MCN are summarized in Table 1.

3.1.2 Numeric Examples

To illustrate the relationship among the three players and the way that revenues are shared, here we provide two numerical examples. In both examples, suppose that x = 7 (i.e., the MCN guarantees to give 70% and 30% of the advertorial fee to the high-type and low-type creators, respectively), the total advertorial fee A = 10,000 (i.e., $10,000), γ = 1 (i.e., the video-sharing platform pays $1 of advertising revenue for each view), ϕ_H = 0.6, and ϕ_L = 0.5 (i.e., the MCN left 60% and 50% of the revenue for the high-type and low-type creators, respectively). The high-type creator's video generates 1000 views while the low-type one's generates 500 views.

In our first example, suppose that the threshold is 800 views. Since the high-type creator meets the threshold, she could get both advertorial fee and advertising revenue, which is $4,800 in total (i.e., $10,000 × 0.7 = $7,000 from the advertorial fee plus $1 × 1,000 = $1,000 from the advertising revenue, and together multiplied by the revenue sharing percentage 60%). As for the low-type creator, since he does not meet the threshold, he gets nothing from the advertorial fee and only receives the advertising revenue, which is $250 in total (i.e., $500 advertising revenue multiplied by the revenue sharing percentage 50%). As for the MCN company, it earns ($7,000 + $1,000) × 0.4 = 3,200 from the high-type creator and $3,000 + $500 × 0.5 = $3,250 from the low-type one. Note that because the low-type creator does not meet the threshold, all the $3,000 goes to the MCN.

As another example, suppose that the threshold is actually 1,500 views. Since both of the creators do not meet the threshold, none of the three players get the advertorial fee. They only receive and share the advertising revenues, which are $600 for the high-type creator, $250 for the low-type creator, and $650 for the MCN company.

3.1.3 The MCN's Optimization Problem

Collectively, after determining the advertorial allocation decision x, the MCN company's contract design problem is to choose ϕ_H and ϕ_L to solve

$$\begin{aligned}\pi_M^I = \max_{\phi_H,\phi_L} \quad & (1-\phi_H)[(a+e_H^I)\beta_H\gamma] \\ & +(1-\phi_L)[(a+e_L^I)\beta_L\gamma] \\ & +\beta_H e_H^I \beta_L e_L^I[(1-\phi_H)xA+(1-\phi_L)(1-x)A] \\ & +\beta_H e_H^I(1-\beta_L e_L^I)[(1-\phi_H)xA+(1-x)A] \\ & +(1-\beta_H e_H^I)\beta_L e_L^I[xA+(1-\phi_L)(1-x)A],\end{aligned}$$

where the two creators solve their effort exertion problems by solving

$$\pi_H^I = \max_{e_H} \phi_H[(a+e_H)\beta_H\gamma+\beta_H e_H xA]-\frac{k}{2}e_H^2$$

and

$$\pi_L^I = \max_{e_L} \phi_L[(a+e_L)\beta_L\gamma+\beta_L e_L(1-x)A]-\frac{k}{2}e_L^2.$$

The sequence of events is depicted in Figure 2. First, the MCN company decides the advertorial allocation proportion for two of the creators, where x is the share for the high-type creator and 1-x is for the low-type one. Second, the MCN company decides the percentages of all revenue, including advertisement revenue and advertorial fee, left for the creators. The revenue percentages left for the high-type and low-type creators are ϕ_H and ϕ_L, respectively. Third, after the contracts are signed, the two creators determine their effort levels, e_H and e_L, independently. The performances of the videos are then observed, the advertisement revenue and advertorial fee are then realized, and all players get paid according to the contracts.

A list of notations is provided in Table 2.

3.2 The Leagued-MCN Structure (L)

Under the leagued-MCN structure, all the settings are similar to the independent-MCN structure except for the ownership of the MCN company. Instead of an independent MCN company existing in the market, the MCN company is co-owned by both high-type and low-type creators. The two creators jointly determine the allocation of the advertorial and the two effort levels. The industry structure of leagued-MCN is depicted in Figure 3.

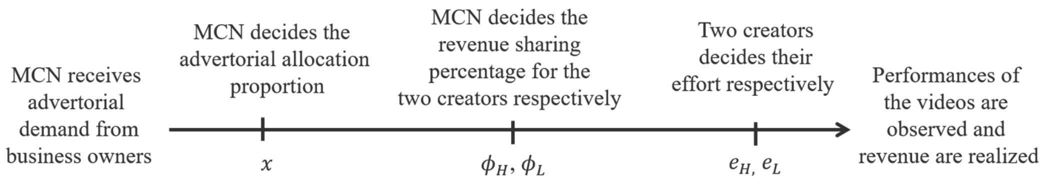

Figure 2 Sequence of Events

Table 2 List of Decision Variables and Parameters

	Decision variables
x	The advertorial proportion allocated to the high-type creator
ϕ_H	The revenue sharing percentage left for the high-type creator
ϕ_L	The revenue sharing percentage left for the low-type creator
e_H	The effort level that the high-type creator makes
e_L	The effort level that the low-type creator makes
	Parameters
β_H	The ability of the high-type creator
β_L	The ability of the low-type creator
a	The natural traffic of views
γ	the per-view payment provided by the sharing platform
A	The total amount of advertorial fee
k	The cost coefficient for creators to make videos (k > 0)

Due to the integrated ownership, both players have the same goal: maximizing the total profit earned by the two players. As a result, the two decision variables previously made by the MCN (the advertorial proportion allocated to the creators x and the revenue sharing percentage left for the creators ϕ_i) disappear, and the two creators together make their effort exertion decisions to solve

$$\pi_{H,L}^{L} = \max_{e_H, e_L} \quad (a + e_H)\beta_H\gamma + (a + e_L)\beta_L\gamma + (\beta_H e_H + \beta_L e_L - \beta_H e_H \beta_L e_L)A - \frac{k}{2}e_H^2 - \frac{k}{2}e_L^2.$$

3.3 The No-MCN Structure (N)

Note that while the leagued-MCN structure may serve as a benchmark of the independent-MCN structure for us to assess the impact of the MCN company, there is another possibility to do the assessment: completely removing the MCN company. Therefore, to better figure out the benefit brought by the MCN company, if any, we consider the third industry structure, the no-MCN structure, which is in between the

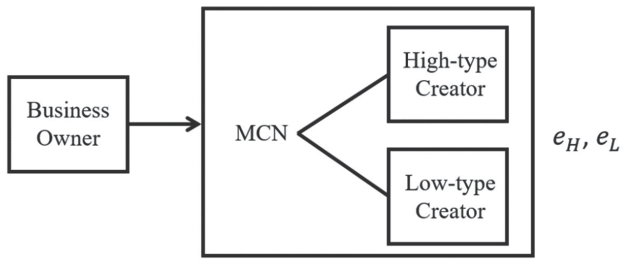

Figure 3 The Leagued-MCN Structure

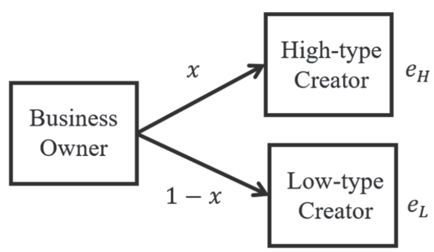

Figure 4 The No-MCN Structure

previous two structures. Under this structure, all the settings are similar to the independent-MCN structure except for the nonexistence of the MCN company. The industry structure of no-MCN is depicted in Figure 4.

Due to the disappearance of the intermediary, business owners who need creators to create advertorials must make the advertorial allocation problem by themselves. We consider one business owner (it) in this model, and it still needs an advertorial to meet a certain performance threshold to achieve the advertising effect. As a result, the business owner's advertorial allocation problem is to maximize the probability of reaching the performance threshold, i.e., to solve

$$\pi_B^N = \max_x \quad \beta_H e_H^N + \beta_L e_L^N - \beta_H e_H^N \beta_L e_L^N.$$

Note that the decision variable x affects e_H^N and e_L^N , which are determined independently by the two creators to maximize their own expected profits

$$\pi_H^N = \max_{e_H} \ (a+e_H)\beta_H\gamma + \beta_H e_H x A - \frac{k}{2}{e_H}^2$$

and

$$\pi_L^N = \max_{e_L} \ (a+e_L)\beta_L\gamma + \beta_L e_L(1-x) A - \frac{k}{2}{e_L}^2.$$

3.4 Assumption

To avoid tedious derivations that do not generate managerial insights, we make some technical assumptions throughout this study in Assumption 1. These assumptions are used for all three MCN-creator structures.

Assumption 1: Let $q_H = \frac{-a\gamma k}{2\beta_H(\gamma+xA)} + \frac{\gamma+A}{2} + \frac{\beta_L A a\gamma}{4(\gamma+(1-x)A)} - \frac{\beta_L^2 A(\gamma+A)}{4k}$ and $q_L = \frac{-a\gamma k}{2\beta_L(\gamma+(1-x)A)} + \frac{\gamma+A}{2} + \frac{\beta_H A a\gamma}{4(\gamma+xA)} - \frac{\beta_H^2 A(\gamma+A)}{4k}$. We assume that $\beta_H\beta_L A < 2k$, $q_H \geq 0$, $q_L \geq 0$, $4k\beta_H^2 q_H \leq 4k^2 - \beta_H^2\beta_L^2 A^2$, $4k\beta_L^2 q_L \leq 4k^2 - \beta_H^2\beta_L^2 A^2$, $4k^2 q_H \leq (4k^2 - \beta_H^2\beta_L^2 A^2)(\gamma + xA)$, and $4k^2 q_L \leq (4k^2 - \beta_H^2\beta_L^2 A^2)(\gamma + (1-x)A)$.

These technical assumptions may be categorized into three groups according to their major implications. In particular, $\beta_H \beta_L A < 2k$, $q_H \geq 0$, and $q_L \geq 0$ are to make the MCN's profit function concave, $4k\beta_H^2 q_H \leq 4k^2 - \beta_H^2\beta_L^2 A^2$ and $4k\beta_L^2 q_L \leq 4k^2 - \beta_H^2\beta_L^2 A^2$ are to make the probabilities for the creators to meet the target $\beta_H e_H$ and $\beta_H e_H$ no greater than 1 in equilibrium, and the last two conditions are to make the revenue sharing ratios ϕ_H and ϕ_L no greater than 1 in equilibrium.

4. Equilibrium Analysis for the Independent-MCN Structure (Structure I)

We first analyze the interaction of the three players under the independent-MCN structure by backward induction. Once we characterize the equilibrium decisions, we interpret the results and obtain managerial implications.

4.1 Creators' Effort Exertion under Structure I

First, Given the allocation decision x and revenue sharing percentages ϕ_H and ϕ_L, we derive the two players' optimal effort levels, e_H^I and e_L^I.

Proposition 1: The optimal effort levels that creators should make are

$$e_H^I = \frac{\phi_H\beta_H(\gamma+xA)}{k} \quad \text{and} \quad e_L^I = \frac{\phi_L\beta_L[\gamma+(1-x)A]}{k}. \tag{1}$$

Moreover, we have e_i^I increases in ϕ_i, β_i, γ, A and decreases in k, where $i \in \{H,L\}$. Besides, e_H^I increases in x; while e_L^I decreases in x.

It is shown that each of the two effort levels is increasing in the revenue sharing percentage ϕ_i , the creator's ability β_i , the per-view payment γ provided by the sharing platform, the amount of the advertorial fee A, and the guaranteed proportion of the advertorial fee. In other words, the MCN company may incentivize a creator to work harder by allocating more advertorial fees to her/him or leaving a larger share of total revenue to her/him.

4.2 MCN Company's Contract Design under Structure I

Second, by predicting how the revenue sharing percentages and advertorial allocation proportion may affect the creators' effort levels, the MCN company maximizes its expected profit by determining the revenue sharing percentages as functions of the advertorial allocation proportion. Note that as the two creators' success or not jointly affect the MCN company's profit, the two revenue sharing percentages must be determined simultaneously. By characterizing a necessary and sufficient condition for the profit function to be jointly concave, we derive the optimal revenue sharing percentages ϕ_H^I and ϕ_L^I in closed forms.

Proposition 2: The optimal revenue percentages left for the two creators are

$$\phi_H^I = \frac{4k^2 q_H}{(4k^2 - \beta_H^2 \beta_L^2 A^2)(\gamma + xA)} \text{ and } \phi_L^I = \frac{4k^2 q_L}{(4k^2 - \beta_H^2 \beta_L^2 A^2)(\gamma + (1-x)A)}, \tag{2}$$

where q_H and q_L are defined in Assumption 1. Moreover, we have ϕ_H^I increases in β_H , and ϕ_L^I increases in β_L.

It is demonstrated that when a creator's ability β_i is improved, the revenue sharing percentage ϕ_i^I left for her/him will also increase. This is because the creator's effort level is affected by her/his ability and the revenue sharing percentage in a multiplicative form, and thus increasing the revenue sharing percentage is more effective when the creator has a higher ability.

It should also be noted that Proposition 2 is based on the condition $\beta_H \beta_L A < 2k$ (cf. Assumption 1). If this condition does not hold, i.e., the cost of effort exertion k is too small, the MCN will find it optimal to induce the creators to exert an infinitely large amount of efforts. As this is not realistic in practice, we impose a technical assumption to eliminate this possibility.

4.3 MCN Company's Advertorial Allocation under Structure I

The final part of our analysis is on the advertorial allocation decision. Unfortunately, due to the great complexity of the problem, we are unable to derive the optimal advertorial allocation proportion x^I in a closed-form expression. We thus resort to extensive numerical studies to observe the impact of adjusting the proportion.

Our first finding is that if $\gamma \neq 0$, the optimal proportion x^I may be 1, between 0 and 1, but is never 0. In other words, while it is possible to allocate all the advertorial fee to the high-type creator or split the fee for both creators, the MCN company should never allocate all the advertorial to the low-type creator when both of the two revenue sources (advertising revenue from a video sharing platform and advertorial fee revenue from a business owner) exist. This is because the ability of low-type creator is weaker than the high-type one, and if the MCN company finds it optimal to put all its resources to one single creator, it is better to delegate to the stronger one, leaving the weaker one to focus on advertising revenue that does not require a threshold.

We also examine the effect of the following exogenous variables to the MCN company's allocation decision. First, we are particularly interested in whether the MCN company should adopt the splitting strategy, which gives the advertorial fee to the two creators separately, or the focusing strategy, which allocates resource only to the high-type creator. Figure 5 depicts the relationship between the two creators' abilities and the optimal allocation strategy, where the horizontal and vertical axes are the high-type and low-type creators' abilities, respectively. It shows that the splitting strategy is optimal if and only if the abilities of these two creators are close enough. This is because when the low-type creator is too weak compared with the high-type one, allocating any resource to her/him is inefficient.

Regarding the production cost, the idea is roughly the same. When the cost is quite high, allocating anything to the low-type creator has a tiny effect on increasing the effort level. When the cost is quite low, allocating everything to the high-type creator is good enough. The splitting strategy is optimal if and only if the cost is neither too high nor too low.

Figure 6 depicts the relationship between the amount of advertorial, the cost of producing videos, and the optimal allocation strategy. It may be observed that the splitting strategy is optimal if any of the two factors are moderate, while the focusing strategy is optimal if any of them are extreme. Consider the advertorial fee first. When the fee is too small, splitting it will make each part too small to incentivize any creator. On the contrary,

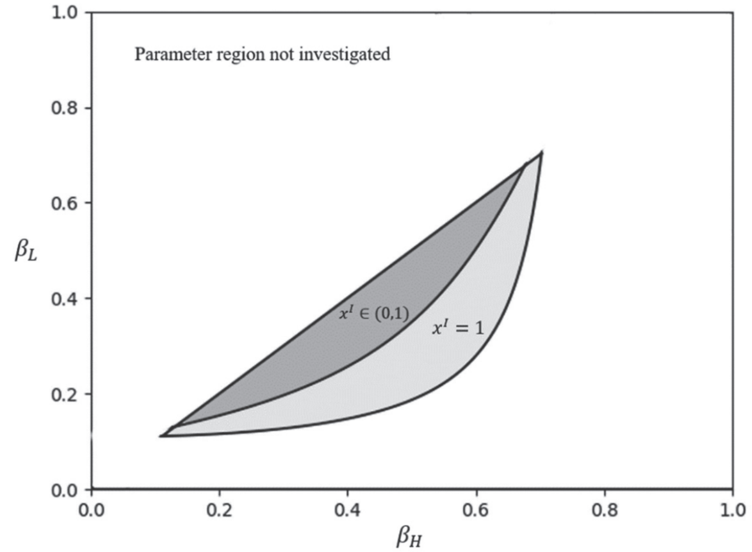

Figure 5 Impact of the Creators' Abilities on the Advertorial Allocation Proportion

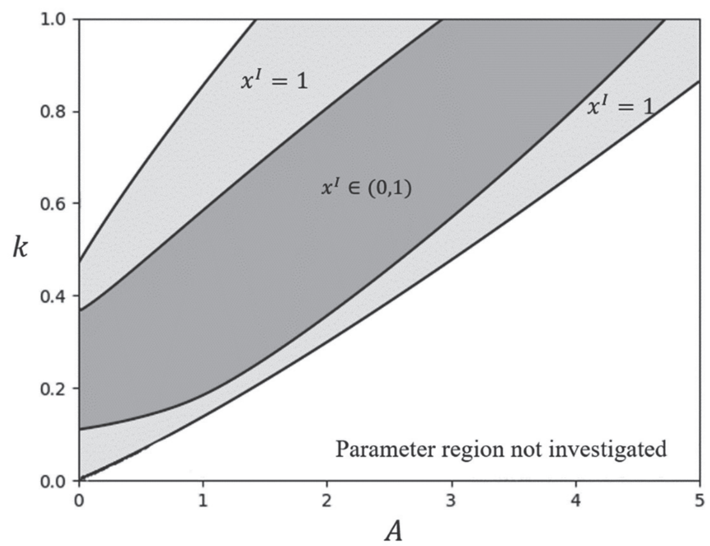

Figure 6 Impact of the Advertorial Fee and Production Cost on Advertorial Allocation

when it is large enough, allocating all of it to the high-type creator makes the overall probability of reaching the threshold high enough, and the low-type creator may also be given up. Only when the fee is moderate should the MCN company split it to make it somewhat possible for both creators to meet the threshold. Regarding the production cost, the idea is roughly the same. When the cost is quite high, allocating anything to the low-type creator has a tiny effect on increasing the effort level. When the cost is quite low, allocating everything to the high-type creator is good enough. The splitting strategy is optimal if and only if the cost is neither too high nor too low.

5. Comparison among the Three Structures

5.1 The Independent-MCN Structure (I) versus the No-MCN One (N)

To figure out how the existence of the MCN (the intermediary) would affect the business environment, we compare structure I and structure N regarding the advertorial allocation decision.

Proposition 3: The optimal advertorial allocation decision under structure N is

$$x^N = 1 \geq x^I. \tag{3}$$

It is shown that under structure N, the advertorial would always wholly be allocated to the high-type creator, and the low-type creator would never get the advertorial. However, under structure I, the low-type creator may get partial advertorial under certain circumstances. It is because the MCN cares about maximizing the profit it gets from the advertorial revenue sharing, whereas the business owner cares only about maximizing the probability of reaching the threshold. Since the revenue sharing percentage that the MCN gets from the low-type creator is usually larger than the one from the high type, the MCN may want to allocate partial advertorial fees to the low-type creator under certain circumstances. As for the business owner in structure N, allocating all the advertorial to the more capable creator is the only way to maximize the probability of reaching the threshold. The comparison between structures I and N indicates that the existence of the MCN creates more opportunities for weaker creators and thus boosts the diversification of sharing platforms.

One may wonder whether the above finding still holds if the independent MCN does not (or cannot) adopt revenue sharing. In this case, the independent MCN's expected profit function becomes

$$\beta_H e_H^I (1 - \beta_L e_L^I)(1 - x)A + (1 - \beta_H e_H^I)\beta_L e_L^I x A,$$

and straightforward numerical studies show that the optimal x may still be less than 1. In other words, the existence of an independent MCN still benefits weaker creators regardless of the adoption of revenue sharing.

5.2 The Independent-MCN Structure (I) versus the Leagued-MCN One (L)

To see how the ownership of MCN affects the industry, we further compare the independent-MCN structure (I) and the leagued-MCN structure (L) regarding the creators' effort exertion decisions. Here we introduce one additional mild assumption $\beta_H \beta_L A < k$ to avoid tedious derivations. Similar to the original condition $\beta_H \beta_L A < 2k$ (cf. Assumption 1) we impose for structure I, the new condition $\beta_H \beta_L A < k$ is made so that the MCN under structure L does not find it optimal to induce the creators to exert an infinitely large amount of effort level.

Proposition 4: There exists a threshold $\gamma_0 > 0$ such that when $\gamma < \gamma_0$ and

$$\frac{3\beta_H^2 A - \sqrt{9\beta_H^4 A^2 - 8\beta_H^2 \beta_L^2 A^2}}{4} < k < \frac{3\beta_H^2 A + \sqrt{9\beta_H^4 A^2 - 8\beta_H^2 \beta_L^2 A^2}}{4}, \text{ we have } e_L^I > e_L^L.$$

It is demonstrated that when the per-view payment γ provided by the sharing platform is low enough, the effort level of the low-type creator under structure I may be larger than the one under structure L. It is because under structure I, the independent MCN usually takes away a larger proportion of the revenue from the low-type creator,[5] which sometimes drives it to allocate partial advertorial to the low-type creator. This decision further motivates the low-type creator to increase his effort level. As for structure L, in order to maximize the total profit, the creator with a higher ability would take charge of more tasks since she is more likely to achieve the threshold. When γ is small, which means that the creators' effort exertion decisions depend mostly on the revenue from the advertorial, the impact of the advertorial allocation decision on the low-type creator's effort exertion decision becomes more significant, making the effort of low-type creator under structure I possible to be larger than the one under structure L.

This phenomenon indicates that the existence of the independent MCN company provides certain benefits for the low-type creator. As a result, the low-type creator may prefer the independent-MCN industry structure rather than the leagued-MCN structure. Since the construction of the leagued-MCN structure requires the agreement between both the high- and low-type creators, and the low-type creator tends to prefer structure I, structure L is hard to be realized in reality. This also provides illuminating justification

5 From Proposition 2, we know that when a creator's ability β_i is improved, the revenue sharing percentage ϕ_i^I left for her/him will also increase. Therefore, the lower the creator's ability is, the higher the proportion of revenue MCN can take away.

for the reason why the independent-MCN industry structure, instead of the leagued-MCN structure is broadly practiced in the real world.

One may argue that the difference between the independent-MCN structure and the leagued-MCN structure contains two aspects. Hence, we offer an alternative structure: the integrated-creator structure. Under this structure, all the settings are the same as the independent-MCN structure except that the two creators would make their effort decisions together, which is much closer to structure L. The industry structure of the integrated-creator structure is depicted in Figure 7. We find that since the profit maximization problem for the creators is simply the addition of the two creators' profits in structure I, the optimal results are exactly the same as those in structure I. In other words, the integrated-creator structure is actually the same as the independent-MCN structure.

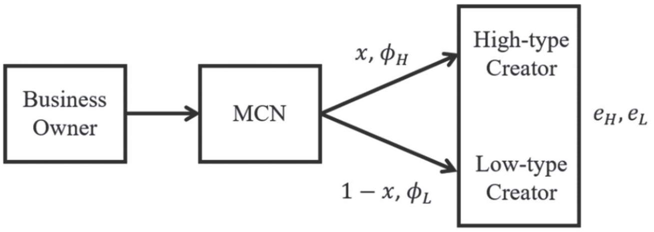

Figure 7 The Integrated-Creator Structure

6. Extensions

In this section, we relax some of our model assumptions to examine the robustness of our major findings and generate new managerial insights.

6.1 Dependency of Meeting the Threshold

In our basic model, we assume that whether a creator's video meets the threshold is independent of whether the other one does. As this may not be true in some cases, here we extend our model setting to see whether our conclusions delivered above are still valid when the two random events are dependent. To facilitate discussion, we will use p and q as the marginal probabilities for the type-H and type-L creators to meet the threshold, P_{HL} , P_H , P_L, and P_0 as the joint probabilities for both creators, and only the type-H creator, only the type-L creator, and no creator to meet the threshold.

Recall that in our basic model, the marginal probabilities for the type-H and type-L creators to meet the threshold are $p = \beta_H e_H$ and $q=\beta_L e_L$, respectively. When the two events are independent, the joint probability for both creators to meet the threshold P_{HL} should be $pq=\beta_H e_H \beta_L e_L$. To make the two events dependent while maintaining the marginal probabilities unchanged, we now assume that $P_{HL} = pq + \eta$, $P_H=p(1-q)-\eta$, $P_L=(1-p)q-\eta$, and $P_0=(1-p)(1-q)+\eta$, where η is an exogenous parameter measuring the dependency between the two creators' videos. If the parameter η is positive, the two events are positively correlated (maybe because the two creators' styles are similar). On the contrary, if η is negative, the two events are negatively correlated. Note that when $\eta = 0$, the two events are independent, and the model is the same as our basic model. Table 3 lists the marginal and joint probabilities for all possible events.

Table 3 The Joint Probability Table under Dependency

Type-H creator	Type-L creator		Marginal probability
	Meets	Does not meet	
Meets	$P_{HL} = pq + \eta$	$P_H = p(1 - q) - \eta$	p
Does not meet	$P_L = (1 - p)\, q - \eta$	$P_0 = (1 - p)\,(1 - q) + \eta$	1-p
Marginal probability	q	$1 - q$	

We analyze the new setting results in the following proposition. It turns out that the degree of dependency does not affect any player's decision.

Proposition 5: For any reasonable value of η, the equilibrium effort exertion, contract design, and advertorial allocation decisions are all identical.

With the new setting of probabilities, the two creators' effort exertion problems remain unchanged. This is because each of them only cares about whether her/himself may meet the threshold, and the marginal probabilities remain unchanged. It then follows that the equilibrium effort levels will still be those derived in Proposition 1. More interestingly, though the MCN company's contract design problem is changed, the MCN company's decisions remain unchanged. To explain this, let's say $\eta > 0$ for a while. In this case, the additional benefit that may be earned when both creators meet the threshold is offset by the additional loss that will happen when only one creator meets the threshold with only an exogenous amount ηA deducted from the MCN company's total profit. The opposite happens if $\eta < 0$. It then follows that all our major findings remain valid with dependency between the two events (except that the MCN company's profit will be affected by an exogenous amount).

6.2 Difference in the Effort Exerting Costs

In our basic model, it is assumed that the effort exertion cost is $\frac{k}{2}e^2$ for both creators, where e is the effort level and k is a common exogenous parameter. With this assumption, the two creators are identical in the cost of exerting efforts and different only in their attractiveness (modeled with the two different values of β_H and β_L). It is admittedly true that in some cases the two creators may also be different in their effort exertion costs. To model this, we now assume that the effort exertion cost is $\frac{k_H}{2}e^2$ for the type-H creator and $\frac{k_L}{2}e^2$ for the type-L creator, where $k_H \neq k_L$. Note that we do not assume $k_H > k_L$ or $k_H < k_L$ as either way is possible.

Under this extended setting, the analysis may still be done following the same backward induction procedure we used for our basic model (except that the technical condition $\beta_H \beta_L A<2k$ for the homogeneous cost case is replaced by $\beta_H\beta_L A < 2\sqrt{k_H k_L}$). With the equilibrium revenue sharing ratios derived, the following proposition helps us understand how the new cost coefficients affect the MCN company's choice of the revenue sharing ratios, for which we say $\phi_H^I(k_H, k_L)$ is for the type-H creator and $\phi_L^I(k_H, k_L)$ is for the type-L one.

Proposition 6: Suppose that $\beta_H\beta_L A < 2\sqrt{k_H k_L}$. Given any values of k_H and k_L such that $k_H \neq k_L$, $\phi_H^I(k_H, k_L)$ increases in k_L and decreases in k_H, and $\phi_L^I(k_H, k_L)$ increases in k_H and decreases in k_L.

According to Proposition 6, the revenue sharing ratio for a creator decreases in the creator's cost coefficient but increases in that of the other creator. The intuition is the following. Let's say the cost coefficient of the type-H creator, k_H, has increased. This is going to drive the type-H creator to exert a lower effort and decrease the probability for her to meet the threshold. As the MCN company realizes that the efficiency of sharing revenue to induce a high effort level is reduced, its optimal response is to cut down $\phi_H^I(k_H, k_L)$ to avoid giving out some inefficient share to the type-H creator. On the contrary, it becomes relatively easier for the MCN company to induce the type-L creator to exert a high effort. The revenue sharing ratio $\phi_L^I(k_H, k_L)$ should thus be increased to capture the additional efficiency.

It is interesting to compare Proposition 6 with Proposition 2. While ϕ_H^I and ϕ_L^I are derived in Proposition 2 by assuming $k_H = k_L = k$, the first-order derivatives of ϕ_H^I and ϕ_L^I with respect to k are quite messy and do not generate insights regarding how the cost coefficient affects the revenue sharing ratios. By splitting the cost coefficient into two distinct values, our analysis in Proposition 6 indirectly reveals the two opposite forces that a common cost coefficient k may generate on the revenue sharing ratios. This explains why k does not have a monotone impact on the revenue sharing ratios in our basic model.

The impact of the two cost coefficients k_H and k_L on the advertorial allocation decision x is also worth investigating. As it is again too complicated to obtain analytical solutions, we resort to numerical solutions to extend our analysis previously done with Figure 6. In Figure 8, we fix the advertorial fee A to see how the two cost coefficients affect advertorial allocation. Similar to the implication from Figure 6, the focusing strategy is optimal when parameter values are extreme, and the splitting strategy is optimal when parameter values are moderate. Note that the best strategy is to allocate the whole advertorial to the high-type creator when k_L is small but k_H is large. This somewhat surprising result can be explained as follows. When k_H is large, the high-type creator does not want to exert a lot of effort. In this case, the MCN must find a way to incentivize the high-type creator, who is more likely to meet the target and win the advertorial fee for the MCN. The MCN thus finds it optimal to allocate as much as possible to the high-type creator.

6.3 Binary Advertorial Allocation

Lastly, we discuss the impact of the flexibility of advertorial allocation. In this study, we assume that the MCN company may split the advertorial fee and assign fractions to

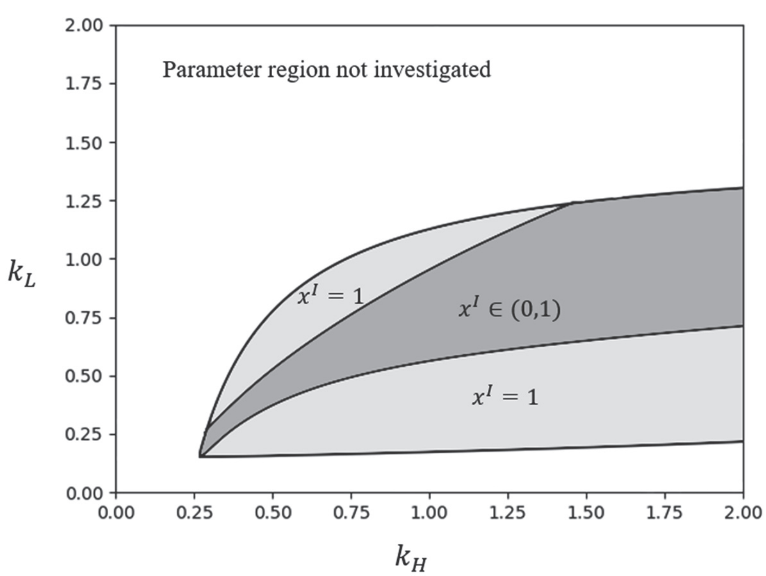

Figure 8 Impact of the Two Production Costs on Advertorial Allocation

multiple creators, and it is natural to ask what will happen if this is not allowed. In this case, the advertorial allocation decision is said to be binary, i.e., either to the type-H creator or the type-L one. Technically, the setting $x \in [0,1]$ in the basic model is changed to $x \in \{0,1\}$, where $x = 1$ means allocating the advertorial to the type-H creator.

To understand the impact of this binary restriction, note that the derivations in Sections 4.1 and 4.2 need no change. However, the fact obtained in Section 4.3 that the optimal proportion x^I may be 1, between 0 and 1, but is never 0 if $\gamma \neq 0$, should be modified. As x can only be 0 or 1 now, the MCN company will always find it optimal to allocate the advertorial to the type-H creator (i.e., $x^I = 1$ for sure). The comparison between the independent-MCN and no-MCN structures in Proposition 3 will also be changed to $x^N = 1 = x^I$. In other words, the low-type creator will have no chance to get allocated an advertorial, and the benefit of having an independent MCN to protect and help weak creators will disappear. Though splitting an advertorial to multiple creators is currently not a popular practice (in fact, no news article or report is found to have reported the wide application of this practice), our analysis suggests MCN companies (or the community) consider this to help create a more diverse ecosystem for video sharing and digital content creation.

7. Conclusions

In this study, we address the advertorial allocation and contract design problem of an MCN company by taking the creators' different abilities and effort exertion decisions into consideration. Besides, we compare the advertorial allocation decisions and the effort exertion decisions under different types of industry structures. Through analytical modeling and analysis, we characterize the equilibrium effort levels and revenue sharing percentages given any predetermined advertorial allocation decision and then make comparisons among different industry structures. It is indeed true that the MCN company may incentivize a creator to exert more effort by allocating a larger proportion of an advertorial to her/him, and the revenue sharing percentages should be adjusted according to the allocation proportion. Regarding the allocation proportion, we identify several factors that affect the adoption of the focusing strategy or splitting strategy. In particular, the splitting strategy is optimal if and only if the creators' abilities are similar, the advertorial fee is moderate, and the cost of making videos is moderate. Our findings may shed light on MCN companies in practice to consider how to operate their companies.

Besides, our models suggest that the existence of an independent MCN gives weaker creators more opportunities regarding advertorial business. As a result, we could infer that MCN companies may facilitate the diversity of the video-sharing industry and thus enrich the whole ecosystem. Consequently, it seems that the rise of MCN companies is beneficial to video-sharing platforms and that the platforms should adopt a positive attitude towards the trend. Indeed, in the real business world, the largest video-sharing platform, YouTube, has adopted several methods to recognize and assist the cooperation between MCN companies and creators. On the one hand, although YouTube does not endorse the service provided by MCN companies, it offers creators the instruction of MCN companies.[6] Moreover, YouTube provides certain functions to support the cooperation between MCN companies and creators, such as sending creators' income directly to MCN companies' accounts and sharing information about creators' performances with MCN companies. On the other hand, YouTube sets policies for MCN companies and lists those MCN companies who have direct relationships with YouTube on its service directory.[7] Through the list, creators could find out whether an MCN company is in touch with YouTube on a regular basis,[8] making themselves easier to reach a reliable MCN company. The findings from our study may explain the above practices and provide valuable insights for video sharing platforms, such as YouTube, to determine their further actions towards MCN companies.

Our study has its limitations. First, so far, all our findings regarding advertorial allocation are based on numerical studies. It would be nice if one may modify our model to obtain some analytical findings. Second, in practice, an MCN company may group multiple creators to make one video together. This not only increases the probability of meeting a certain number of views but also helps a creator with lower popularity to become famous. It will be interesting if such a strategic decision may be further explored.

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6 See https://support.google.com/youtube/answer/2737059?hl=en

7 See https://servicesdirectory.withyoutube.com/

8 See https://servicesdirectory.withyoutube.com/directory/#?services=multi-channel-networks

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Appendix

Proof of Proposition 1. Using backward induction, we solve the second stage first. The first- and second-order derivatives of $\pi_H^I(e_H)$ with respect to e_H are

$$\frac{\partial \pi_H^I(e_H)}{\partial e_H} = \phi_H \beta_H(\gamma + xA) - ke_H \text{ and } \frac{\partial^2 \pi_H^I(e_H)}{\partial e_H^2} = -k,$$

respectively. Since $k > 0$, we have $\frac{\partial^2 \pi_H^I(e_H)}{\partial e_H^2} < 0$; the function is concave. Due to concavity, the optimal solution must satisfy $\frac{\partial \pi_H^I(e_H)}{\partial e_H} = 0$. By solving the equation, we obtain e_H^I as stated in (1). Similarly, those derivatives of $\pi_L^I(e_L)$ with respect to e_L are

$$\frac{\partial \pi_L^I(e_L)}{\partial e_L} = \phi_L \beta_L(\gamma + (1-x)A) - ke_L \text{ and } \frac{\partial^2 \pi_L^I(e_L)}{\partial e_L^2} = -k.$$

Since $k > 0$, we have $\frac{\partial^2 \pi_L^I(e_L)}{\partial e_L^2} < 0$; the function is concave. Due to concavity, the optimal solution must satisfy $\frac{\partial \pi_L^I(e_L)}{\partial e_L} = 0$. By solving the equation, we obtain e_L^I as stated in (1).

Since $\frac{\partial e_H^I}{\partial \phi_H} = \frac{\beta_H(\gamma+xA)}{k} > 0$ and $\frac{\partial e_L^I}{\partial \phi_L} = \frac{\beta_L[\gamma+(1-x)A]}{k} > 0$, we have e_i^I increases in ϕ_i. Since $\frac{\partial e_H^I}{\partial \beta_H} = \frac{\phi_H(\gamma+xA)}{k} > 0$ and $\frac{\partial e_L^I}{\partial \beta_L} = \frac{\phi_L[\gamma+(1-x)A]}{k} > 0$, we have e_i^I increases in β_i. Since $\frac{\partial e_H^I}{\partial \gamma} = \frac{\phi_H \beta_H}{k} > 0$ and $\frac{\partial e_L^I}{\partial \gamma} = \frac{\phi_L \beta_L}{k} > 0$, we have e_i^I increases in γ. Since $\frac{\partial e_H^I}{\partial A} = \frac{\phi_H \beta_H x}{k} > 0$ and $\frac{\partial e_L^I}{\partial A} = \frac{\phi_L \beta_L(1-x)}{k} > 0$, we have e_i^I increases in A. Since

$\frac{\partial e_H^I}{\partial k} = -\frac{\phi_H \beta_H(\gamma+xA)}{k^2} < 0$ and $\frac{\partial e_L^I}{\partial k} = -\frac{\phi_L \beta_L[\gamma+(1-x)A]}{k^2} < 0$, we have e_i^I decreases in k. Since $\frac{\partial e_H^I}{\partial x} = \frac{\phi_H \beta_H A}{k} > 0$ and $\frac{\partial e_L^I}{\partial x} = \frac{-\phi_L \beta_L A}{k} < 0$, we have e_H^I increases in x and e_L^I decreasing in x. Q.E.D.

Proof of Proposition 2. We plug in e_H^I and e_L^I into π_M^I and get

$$\begin{aligned}\pi_M^I(\phi_H, \phi_L) = \quad & (1-\phi_H)\left[a\beta_H\gamma + \frac{\phi_H\beta_H^2\gamma(\gamma+xA)}{k}\right] \\ & +(1-\phi_L)\left[a\beta_L\gamma + \frac{\phi_L\beta_L^2\gamma(\gamma+(1-x)A)}{k}\right] \\ & +\frac{\beta_H^2\beta_L^2\phi_H\phi_L(\gamma+xA)(\gamma+(1-x)A)[(1-\phi_H)xA+(1-\phi_L)(1-x)A]}{k^2} \\ & +\frac{\beta_H^2\phi_H(\gamma+xA)[k-\beta_L^2\phi_L(\gamma+(1-x)A)]A(1-\phi_H x)}{k^2} \\ & +\frac{\beta_L^2\phi_L(\gamma+(1-x)A)[k-\beta_H^2\phi_H(\gamma+xA)]A\big(1-\phi_L(1-x)\big)}{k^2}.\end{aligned}$$

We then have $\frac{\partial^2 \pi_M^I}{\partial \phi_H^2} = \frac{-2\beta_H^2(\gamma+xA)^2}{k} < 0$, the Hessian matrix of π_M^I being

$$\begin{bmatrix} \frac{-2\beta_H^2(\gamma+xA)^2}{k} & \frac{-\beta_H^2\beta_L^2A(\gamma+xA)(\gamma+(1-x)A)}{k^2} \\ \frac{-\beta_H^2\beta_L^2A(\gamma+xA)(\gamma+(1-x)A)}{k^2} & \frac{-2\beta_L^2(\gamma+(1-x)A)^2}{k} \end{bmatrix},$$

and the determinant of the Hessian matrix being

$$\frac{\beta_H^2\beta_L^2(\gamma+xA)^2(\gamma+(1-x)A)^2}{k^2}\left(4-\frac{\beta_H^2\beta_L^2A^2}{k^2}\right).$$

As we assume that $\beta_H \beta_L A < 2k$ in Assumption 1, the determinant is positive, and thus π_M^I is jointly concave in ϕ_H and ϕ_L. Due to concavity, the optimal solution must satisfy $\frac{\partial \pi_M^I}{\partial \phi_H} = 0$ and $\frac{\partial \pi_M^I}{\partial \phi_L} = 0$, where

$$\begin{aligned}\frac{\partial \pi_M^I}{\partial \phi_H} = \; & \frac{\beta_H^2 \gamma(\gamma + xA)}{k} - a\beta_H \gamma - \frac{2\phi_H \beta_H^2 \gamma(\gamma + xA)}{k} \\ & - \frac{2\beta_H^2 \beta_L^2 \phi_H \phi_L (\gamma + xA)(\gamma + (1-x)A)Ax}{k^2} \\ & + \frac{\beta_H^2 (\gamma + xA)[k - \beta_L^2 \phi_L (\gamma + (1-x)A)]A}{k^2} \\ & - \frac{2\beta_H^2 \phi_H (\gamma + xA)[k - \beta_L^2 \phi_L (\gamma + (1-x)A)]Ax}{k^2}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial \pi_M^I}{\partial \phi_L} = \; & \frac{\beta_L^2 \gamma(\gamma + (1-x)A)}{k} - a\beta_L \gamma - \frac{2\phi_L \beta_L^2 \gamma(\gamma + (1-x)A)}{k} \\ & - \frac{2\beta_H^2 \beta_L^2 \phi_H \phi_L (\gamma + xA)(\gamma + (1-x)A)A(1-x)}{k^2} \\ & + \frac{\beta_L^2 (\gamma + (1-x)A)[k - \beta_H^2 \phi_H (\gamma + xA)]A}{k^2} \\ & - \frac{2\beta_L^2 \phi_L (\gamma + (1-x)A)[k - \beta_H^2 \phi_H (\gamma + xA)]A(1-x)}{k^2}.\end{aligned}$$

By solving the system of equations, we obtain ϕ_H^I and ϕ_L^I as stated in (2). Given Assumption 1, ϕ_H^I and ϕ_L^I are both positive and thus feasible.

The derivatives of ϕ_H^I with respect to β_H is

$$\frac{\partial \phi_H^I}{\partial \beta_H} = \frac{8k^2 \beta_H \beta_L^2 A^2 q_H}{(4k^2 - \beta_H^2 \beta_L^2 A^2)^2 (\gamma + xA)} + \frac{4k^2}{4k^2 - \beta_H^2 \beta_L^2 A^2}\left[\frac{a\gamma k}{2\beta_H^2 (\gamma + xA)^2}\right].$$

According to Assumption 1, $q_H \geq 0$ and $\beta_H\beta_L A < 2k$. Thus, we have $4k^2 - \beta_H^2\beta_L^2A^2 > 0$ and thus $\frac{\partial\phi_H^I}{\partial\beta_H} > 0$, which means that ϕ_H^I increase in β_H. Similarly, the derivatives of ϕ_L^I with respect to β_L is

$$\frac{\partial\phi_L^I}{\partial\beta_L} = \frac{8k^2\beta_H^2\beta_L A^2 q_L}{(4k^2 - \beta_H^2\beta_L^2A^2)^2(\gamma + (1-x)A)} + \frac{4k^2}{4k^2 - \beta_H^2\beta_L^2A^2}\left[\frac{a\gamma k}{2\beta_L^2(\gamma + (1-x)A)^2}\right].$$

According to Assumption 1, $q_L \geq 0$ and $\beta_H\beta_L A < 2k$. Thus, we have $4k^2 - \beta_H^2\beta_L^2A^2 > 0$ and get $\frac{\partial\phi_L^I}{\partial\beta_L} > 0$, which means that ϕ_L^I increase in β_L. Q.E.D.

Proof of Proposition 3. Using backward induction, we solve the second stage first. The first- and second-order derivatives of $\pi_H^N(e_H)$ with respect to e_H are

$$\frac{\partial\pi_H^N(e_H)}{\partial e_H} = \beta_H(\gamma + xA) - ke_H \text{ and } \frac{\partial^2\pi_H^N(e_H)}{\partial e_H^2} = -k,$$

respectively. Since $k > 0$, we have $\frac{\partial^2\pi_H^N(e_H)}{\partial e_H^2} < 0$; the function is concave. Due to concavity, the optimal solution must satisfy $\frac{\partial\pi_H^N(e_H)}{\partial e_H} = 0$. By solving the equation, we obtain $e_H^N = \frac{\beta_H(\gamma+xA)}{k}$. Similarly, those derivatives of $\pi_L^N(e_L)$ with respect to e_L are

$$\frac{\partial\pi_L^N(e_L)}{\partial e_L} = \beta_L(\gamma + (1-x)A) - ke_L \text{ and } \frac{\partial^2\pi_L^N(e_L)}{\partial e_L^2} = -k.$$

Since $k > 0$, we have $\frac{\partial^2\pi_L^N(e_L)}{\partial e_L^2} < 0$; the function is concave. Due to concavity, the optimal solution must satisfy $\frac{\partial\pi_L^N(e_L)}{\partial e_L} = 0$. By solving the equation, we obtain $e_L^N = \frac{\beta_L(\gamma+(1-x)A)}{k}$. We then plug in e_H^N and e_L^N into π_B^N and get

$$\pi_B^N(x) = \frac{\beta_H^2(\gamma + xA)}{k} + \frac{\beta_L^2(\gamma + (1-x)A)}{k} - \frac{\beta_H^2(\gamma + xA)\beta_L^2(\gamma + (1-x)A)}{k^2}.$$

Since $\pi_B^N(x)$ is a quadratic function of x and the coefficient of x^2 is $\frac{\beta_H^2\beta_L^2A^2}{k^2} > 0$, the parabola opens upward. As a result, we know that the maximum of $\pi_B^N(x)$ exists either when x is the minimum or the maximum, namely $x = 0$ or $x = 1$. We then plug in $x = 0$ and $x = 1$ into π_B^N respectively and get

$$\pi_B^N(1) = \frac{\beta_H^2(\gamma + A)}{k} + \frac{\beta_L^2\gamma}{k} - \frac{\beta_H^2\beta_L^2(\gamma + A)}{k^2}$$

$$\pi_B^N(0) = \frac{\beta_H^2\gamma}{k} + \frac{\beta_L^2(\gamma + A)}{k} - \frac{\beta_H^2\beta_L^2(\gamma + A)}{k^2}$$

Since $\pi_B^N(1) - \pi_B^N(0) = \frac{A(\beta_H^2 - \beta_L^2)}{k} > 0$, $\pi_B^N(1) > \pi_B^N(0)$, and we obtain x^N as stated in (3). Moreover, since $0 \leq x^I \leq 1$ and $x^N = 1$, the proof is complete. Q.E.D.

Proof of Proposition 4. First, we will find out e_L^L under structure L. We have $\frac{\partial^2 \pi_M^L}{\partial e_H^2} = -k < 0$, the Hessian matrix of π_M^L being

$$\begin{bmatrix} -k & -\beta_H\beta_L A \\ -\beta_H\beta_L A & -k \end{bmatrix},$$

and the determinant of the Hessian matrix being $k^2 - \beta_H^2\beta_L^2A^2$. As we assume that $\beta_H\beta_L A < k$, the determinant is positive, and thus π_M^L is jointly concave in e_H and e_L. Due to concavity, the optimal solution must satisfy $\frac{\partial \pi_M^L}{\partial e_H} = 0$ and $\frac{\partial \pi_M^L}{\partial e_L} = 0$, where

$$\frac{\partial \pi_M^L}{\partial e_H} = \beta_H A - \beta_H \beta_L e_L A + \beta_H \gamma - k e_H$$

and

$$\frac{\partial \pi_M^L}{\partial e_L} = \beta_L A - \beta_H \beta_L e_H A + \beta_L \gamma - k e_L$$

By solving the system of equations, we obtain $e_L^L = \frac{k\beta_L A + k\beta_L \gamma - \beta_H \beta_L A(\beta_H A + \beta_H \gamma)}{k^2 - \beta_H^2 \beta_L^2 A^2}$. If $\gamma = 0$, $e_L^L - e_L^I = \frac{\beta_L k A[2k^2 + \beta_H^2 A(-3k + \beta_L^2 A)]}{(k^2 - \beta_H^2 \beta_L^2 A^2)(4k^2 - \beta_H^2 \beta_L^2 A^2)}$. We know that $2k^2 + \beta_H^2 A(-3k + \beta_L^2 A) < 0$ if $\frac{3\beta_H^2 A - \sqrt{9\beta_H^4 A^2 - 8\beta_H^2 \beta_L^2 A^2}}{4} < k < \frac{3\beta_H^2 A + \sqrt{9\beta_H^4 A^2 - 8\beta_H^2 \beta_L^2 A^2}}{4}$. As we assume that $\beta_H \beta_L A < k$ and $\frac{3\beta_H^2 A - \sqrt{9\beta_H^4 A^2 - 8\beta_H^2 \beta_L^2 A^2}}{4} < k < \frac{3\beta_H^2 A + \sqrt{9\beta_H^4 A^2 - 8\beta_H^2 \beta_L^2 A^2}}{4}$, we have $e_L^L - e_L^I < 0$, namely $e_L^I > e_L^L$.

To show that there exist feasible variable combinations, we take $\beta_H = 0.5$, $\beta_L = 0.2$, $A = 2$, and $k = 0.6$ as an example. It's obvious that all the variables are in feasible regions and $e_L^L - e_L^I < 0$. Q.E.D.

Proof of Proposition 5. As the marginal probabilities remain unchanged, the equilibrium effort decisions are also unchanged. However, because the joint probabilities have been changed, the MCN's contract design problem is changed to

$$\pi_M^I(\eta) = \max_{\phi_H,\phi_L} \quad (1-\phi_H)[(a+e_H^I)\beta_H\gamma] + (1-\phi_L)[(a+e_L^I)\beta_L\gamma]$$
$$+(\beta_H e_H^I \beta_L e_L^I + \eta)[(1-\phi_H)xA+(1-\phi_L)(1-x)A]$$
$$+[\beta_H e_H^I(1-\beta_L e_L^I) - \eta][(1-\phi_H)xA + (1-x)A]$$
$$+[(1-\beta_H e_H^I)\beta_L e_L^I - \eta][xA + (1-\phi_L)(1-x)A],$$

where η enters the three terms regarding advertorial fees. Through arithmetic it may be derived that $\pi_M^I(\eta) = \pi_M^I(0) - \eta A$, where $\pi_M^I(0)$ is exactly π_M^I, the MCN company's profit when the two events are independent. This implies that the MCN company's equilibrium choices of the two revenue sharing ratios ϕ_H and ϕ_L will remain identical regardless of the value of η. Q.E.D.

Proof of Proposition 6. With the new setting, the type-H creator's problem becomes

$$\pi_H^I(k_H, k_L) = \max_{e_H} \ \phi_H[(a+e_H)\beta_H\gamma + \beta_H e_H xA] - \frac{k_H}{2} e_H^2$$

while that of the type-L one becomes

$$\pi_L^I(k_H, k_L) = \max_{e_L} \ \phi_L[(a+e_L)\beta_L\gamma + \beta_L e_L(1-x)A] - \frac{k_H}{2} e_L^2.$$

It is straightforward to see that their equilibrium effort levels will be $e_H^I(k_H) = \frac{\phi_H\beta_H(\gamma+xA)}{k_H}$ *and* $e_L^I(k_L) = \frac{\phi_L\beta_L[\gamma+(1-x)A]}{k_L}$.

The MCN company's problem may then be formulated by replacing e_H^I and e_H^I by $e_H^I(k_H)$ and $e_H^I(k_H)$ in its profit function. By redefining the two parameters q_H and q_L defined in Assumption 1 by $q_H(k_H, k_L) = \frac{-a\gamma k_H}{2\beta_H(\gamma+xA)} + \frac{\gamma+A}{2} + \frac{\beta_L A a\gamma}{4(\gamma+(1-x)A)} - \frac{\beta_L^2 A(r+A)}{4k_L}$ and $q_L = \frac{-a\gamma k_L}{2\beta_L(\gamma+(1-x)A)} + \frac{\gamma+A}{2} + \frac{\beta_H A a\gamma}{4(\gamma+xA)} - \frac{\beta_H^2 A(r+A)}{4k_H}$, the first-order condition yields $\phi_H^I(k_H, k_L) = \frac{4k_H k_L q_H(k_H,k_L)}{(4k_H k_L - \beta_H^2 \beta_L^2 A^2)(\gamma + xA)}$ as the revenue sharing ratio for the type-H creator and $\phi_L^I(k_H, k_L) = \frac{4k_H k_L q_L(k_H,k_L)}{(4k_H k_L - \beta_H^2 \beta_L^2 A^2)(\gamma + (1-x)A)}$ for the type-L one. The first-order derivative of $\phi_H^I(k_H, k_L)$ and $\phi_L^I(k_H, k_L)$ with respective to k_H and k_L then proves the proposition. Q.E.D.

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